Mathematical Economics
Midterm Exam 1

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Name: ____

Instructions:

- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- The exam time is 80 minutes.
- This exam has 6 questions on 5 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use the back of the exam sheet, but make sure to indicate that you are using the back.
- Submit your entire exam sheet before leaving the room, even if some parts are empty or you intend to drop the class.

Question:	1	2	3	4	5	6	Total
Points:	20	20	10	10	20	20	100
Score:							

1. (a) (5 points) Suppose that there are 2 goods, an agent has endowment $e = (e_1, e_2)$, and the price vector is $p = (p_1, p_2)$. Compute the wealth of the agent.

   (b) (5 points) Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ be two vectors in $\mathbb{R}^2$, the 2-dimensional Euclidean space. Give the exact definition for each of the following vector inequalities: $y \geq x$, $y > x$, and $y \gg x$.

   (c) (5 points) What is the definition of a utility function being weakly monotonic?

   (d) (5 points) Consider the sets

$$C = \left\{(x, y) \in \mathbb{R}^2_+ \,\middle|\, xy \geq 6\right\},$$
$$D = \left\{(x, y) \in \mathbb{R}^2_+ \,\middle|\, x < 3, y < 2\right\}.$$

   Can C, D be separated? Answer yes or no. If yes, provide the equation of a separating hyperplane. If no, explain why.

2. Consider an agent with utility function

$$u(x_1, x_2) = 2\sqrt{x_1} + 4\sqrt{x_2}.$$

   Assume that the price is (p_1, p_2) and the agent has wealth w.

   (a) (5 points) Write down the Lagrangian for the utility maximization problem using the Lagrange multiplier $\lambda \geq 0$.

(b) (5 points) Derive the first-order condition with respect to x_1 and x_2.

(c) (5 points) Express the demand x_1, x_2 in terms of λ, p_1, p_2.

(d) (5 points) Solve for the demand (x_1, x_2) using only w, p_1, p_2.

3. Let $\mathcal{E} = \{I, (e_i), (u_i)\}$ be an Arrow-Debreu economy and $\{p, (x_i)\}$ be an equilibrium, where $p = (p_1, \ldots, p_L)$ is the price vector.

   (a) (5 points) Assume $p_l < 0$. If an agent buys $\epsilon > 0$ of extra good l, how much more (or less) will his or her wealth be?

   (b) (5 points) Prove that if we assume free disposal and at least one agent has a locally nonsatiated utility function, then prices cannot be negative.

4. Consider an economy with three goods labeled by $l = 1, 2, 3$. Let p_l be the price of good l. Without loss of generality, set $p_1 = 1$. Suppose an agent has wealth w and utility function

$$u(x_1, x_2, x_3) = x_1 - \frac{1}{2x_2^2} - x_3 \log x_3 + x_3.$$

   (a) (3 points) What is the name of this type of utility function?

   (b) (7 points) Compute the demand of the agent. You may assume that good 1 can be consumed in positive or negative amounts.

5. Consider an economy with two goods, 1 and 2, and two agents, A and B. The utility

function of each agent is

$$u_A(x_1, x_2) = x_1 + 2\sqrt{x_2},$$
$$u_B(x_1, x_2) = x_1 + 2\log x_2.$$

Suppose that the initial endowments are $(e_1^A, e_2^A) = (4, 5)$ and $(e_1^B, e_2^B) = (10, 3)$.

(a) (10 points) Consider the problem of maximizing the sum of utilities,

$$\begin{array}{ll} \text{maximize} & u_A(x_1^A, x_2^A) + u_B(x_1^B, x_2^B) \\ \text{subject to} & x_1^A + x_1^B \le e_1^A + e_1^B, \\ & x_2^A + x_2^B \le e_2^A + e_2^B. \end{array}$$

Let $(x_1^A, x_2^A, x_1^B, x_2^B)$ be a solution. Compute x_2^A and x_2^B.

(b) (10 points) Let the price be $p_1 = 1$ and $p_2 = p$. Compute the equilibrium price p and the allocation.

6. Consider an economy with two goods, 1 and 2, and two agents, A and B. The utility

functions are

$$u_A(x_1, x_2) = \log x_1 + 2 \log x_2,$$
$$u_B(x_1, x_2) = 3x_1 + x_2.$$

The endowments are $(e_1^A, e_2^A) = (6, 3)$ and $(e_1^B, e_2^B) = (10, 50)$.

(a) (3 points) What is the name of the type of agent A's utility function?

(b) (5 points) Let the prices be $p_1 = 1$ and $p_2 = p$. Compute the demand of agent A.

(c) (5 points) Assuming agent B consumes positive amounts of both goods, what is p?

(d) (7 points) Compute the competitive equilibrium.