

# Mathematical Economics

## Midterm Exam 2

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Name: \_\_\_\_\_

Instructions:

- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- The exam time is 80 minutes.
- This exam has 4 questions on 6 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use the back of the exam sheet, but make sure to indicate that you are using the back.
- Submit your entire exam sheet before leaving the room, even if some parts are empty or you intend to drop the class.

Question:	1	2	3	4	Total
Points:	25	20	20	35	100
Score:					

1. Consider an economy with  $L$  goods and  $I$  agents. Agent  $i$  has endowment  $e_i$  and a locally nonsatiated utility function  $u_i(x)$ . Let  $p$  be the price vector. If the notation bothers you, you may set  $L = 2$  and  $I = 2$ .

(a) (5 points) What is the definition of local nonsatiation? You may explain in words (maximum 4 points) or mathematically.

(b) (5 points) Suppose  $x_i$  solves the utility maximization problem

$$\text{maximize } u_i(x) \text{ subject to } p \cdot x \leq p \cdot e_i.$$

Explain why it must be the case that  $p \cdot x_i = p \cdot e_i$ .

(c) (5 points) What does it mean that an allocation  $(y_i)$  Pareto dominates the allocation  $(x_i)$ ? You can explain in words (maximum 4 points) or write down the precise mathematical definition.

(d) (3 points) What does it mean that the feasible allocation  $(x_i)$  is Pareto efficient? You can explain in words.

(e) (7 points) Let  $\{p, (x_i)\}$  be an Arrow-Debreu equilibrium. Prove that  $(x_i)$  is Pareto efficient.

2. Consider an economy with two agents and two goods. The utility functions are

$$u_1(x_1, x_2) = \sqrt{x_1 x_2},$$
$$u_2(x_1, x_2) = \min \{ \sqrt{x_1 x_2}, 7 \}.$$

The initial endowments are  $e_1 = (3, 12)$  and  $e_2 = (12, 3)$ .

(a) (5 points) Show that the price vector  $(p_1, p_2) = (1, 1)$  and the allocation  $x_1 = x_2 = (7.5, 7.5)$  constitute a competitive equilibrium.

(b) (7 points) Show that the equilibrium allocation is Pareto inefficient.

(c) (8 points) Does this example contradict the first welfare theorem? Answer yes or no, and explain why.

3. Consider an economy with two countries,  $i = A, B$ , and two physical goods,  $l = 1, 2$ . The endowment is  $e_A = (6, 1)$  and  $e_B = (1, 6)$ . The utility function is  $u(x_1, x_2) = x_1x_2$  for all agents. Suppose that there are transportation costs, and 50% of the exported goods perish by the time they reach the destination.

(a) (5 points) How many kinds of goods are there in the world? Answer the number and explain the reason.

(b) (5 points) Assuming that country  $A$  imports good 2, what is its price? (Set the price of good 1 equal to 1.)

(c) (10 points) Compute the free trade equilibrium.

4. Consider an economy with three agents ( $i = 1, 2, 3$ ), two goods ( $l = 1, 2$ ), and two countries,  $A, B$ . Agents 1 and 2 live in country  $A$  and agent 3 lives in country  $B$ . The utility functions are

$$u_1(x_1, x_2) = x_1^2 x_2,$$

$$u_2(x_1, x_2) = x_1 x_2^2,$$

$$u_3(x_1, x_2) = x_1 x_2.$$

Suppose that the initial endowments are  $e_1 = e_2 = (3, 3)$  and  $e_3 = (18, 6)$ . In answering questions below, in order to make the notation consistent use  $x_{il}$  for consumption of good  $l$  by agent  $i$ . (So  $x_{12}$  is consumption of good 2 by agent 1, for example.) Also, use  $p_1 = 1$  and  $p_2 = p$  for the prices.

- (a) (5 points) Compute the competitive equilibrium when country  $A$  is in autarky as well as the utility level of each agent.

(b) (10 points) Compute the free trade equilibrium price and allocation.

(c) (5 points) Compute the utility level of each agent and determine who gained from trade and who lost.

- (d) (15 points) Find a tax scheme in country  $A$  such that free trade is Pareto improving. Explain why the tax scheme you suggest is Pareto improving.