

Mathematical Economics

Midterm Exam 2

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Name: _____

Instruction:

- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- The exam time is 80 minutes.
- This exam has 4 questions on 7 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use the back of the exam sheet, but make sure to indicate that you are using the back.
- Submit your entire exam sheet before leaving the room, even if some parts are empty or you intend to drop the class.

Question:	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

2. Consider an economy with two agent with Cobb-Douglas utility functions

$$u_1(x_1, x_2) = \frac{2}{3} \log x_1 + \frac{1}{3} \log x_2,$$
$$u_2(x_1, x_2) = \frac{1}{3} \log x_1 + \frac{2}{3} \log x_2.$$

Assume that the endowments are $e_1 = (1, 2)$ and $e_2 = (1, 3)$.

- (a) (5 points) Consider the allocation (x_1, x_2) (for agent 1) and $(2 - x_1, 5 - x_2)$ (for agent 2). Compute the marginal rate of substitution

$$\frac{\partial u_i}{\partial x_2} / \frac{\partial u_i}{\partial x_1}$$

for each agent $i = 1, 2$ at this allocation.

- (b) (5 points) Show that the initial endowment $\{(1, 2), (1, 3)\}$ is Pareto inefficient.

- (c) (5 points) Show that the allocation $\{(1, 1), (1, 4)\}$ is Pareto efficient.

- (d) (10 points) Compute the price vector $p = (p_1, p_2)$ (with $p_1 = 1$) and transfer payments (t_1, t_2) such that the price p and the allocation $\{(1, 1), (1, 4)\}$ constitute a competitive equilibrium with transfer payments.

3. Consider an economy with two countries, $i = A, B$, and three consumption goods, $l = 1, 2, 3$. Both countries have labor endowment $e_1 = e_2 = 1$. The utility functions are

$$u_A(x_1, x_2, x_3) = \frac{1}{2} \log x_1 + \frac{1}{4} \log x_2 + \frac{1}{4} \log x_3,$$
$$u_B(x_1, x_2, x_3) = \frac{1}{3} \log x_1 + \frac{1}{3} \log x_2 + \frac{1}{3} \log x_3.$$

Each country can produce the consumption goods from labor using the linear technology $y = a_{il}e$, where e is labor input, y is output of good l , and $a_{il} > 0$ is the productivity. Assume that productivities are

$$(a_{A1}, a_{A2}, a_{A3}) = (4, 2, 2),$$
$$(a_{B1}, a_{B2}, a_{B3}) = (1, 1, 2).$$

- (a) (5 points) What is the definition of comparative advantage of country A over B ? Compute the comparative advantage for each industry.
- (b) (5 points) Given the price $p = (p_1, p_2, p_3)$ and the wage w_A of country A , compute the demand of country A .

(c) (5 points) Assuming that both countries produce good 2 in free trade and setting $p_2 = 1$, compute p_1, p_3, w_A, w_B .

(d) (5 points) Compute the free trade equilibrium consumption in each country.

(e) (5 points) Compute the labor allocation across each industry for each country.

4. Consider an economy with three agents ($i = 1, 2, 3$), two goods ($l = 1, 2$), and two countries, A, B . Agents 1 and 2 live in country A and agent 3 lives in country B . The utility function of each agent is

$$u(x_1, x_2) = x_1 x_2.$$

Suppose that the initial endowments are $e_1 = (4, 4)$, $e_2 = (20, 8)$, and $e_3 = (4, 16)$. In answering questions below, in order to make the notation consistent use x_{il} for consumption of good l by agent i . (So x_{12} is consumption of good 2 by agent 1, for example.) Also, use $p_1 = 1$ and $p_2 = p$ for the prices.

- (a) (5 points) Compute the competitive equilibrium when country A is in autarky as well as the utility level of each agent.

- (b) (7 points) Compute the free trade equilibrium price and allocation.

- (c) (3 points) Compute the utility level of each agent and determine who gained from trade and who lost.
- (d) (10 points) Find a tax scheme in country A such that free trade is Pareto improving. Compute the free trade equilibrium after transfer payments and verify that every agent is better off.

