Econ 113 Mathematical Economics
Midterm Exam 1

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Name: ____

Instruction:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, *i.e.*, base $\mathrm{e} = 2.718281828\dots$.
- "Show" is synonymous to "prove".
- Full credit will not be given to correct but unsupported claims. Example: $x^2 - 2x + 1 \ge 0$ is true but not obvious. You need to argue $x^2 - 2x + 1 = (x-1)^2 \ge 0$.
- The exam time is 80 minutes.
- This exam has 5 questions on 5 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	5	Total
Points:	10	30	20	15	25	100
Score:						

1. (a) (5 points) What is the definition of a set being convex? You can explain in words.

   (b) (5 points) What is the definition of a utility function being quasi-linear?

2. Consider an agent with utility function

$$u(x_1, x_2) = 4\sqrt{x_1} + 2\sqrt{x_2}.$$

   Assume that the endowment of the agent is (e_1, e_2) and the price of each good is $p_1 = 1$ and $p_2 = p > 0$.

   (a) (5 points) Compute the wealth w of the agent.

   (b) (5 points) Write down the Lagrangian for the utility maximization problem using the Lagrange multiplier $\lambda \geq 0$. You may use w.

   (c) (5 points) Derive the first-order condition with respect to x_1 and x_2.

   (d) (5 points) Express the demand x_1, x_2 in terms of λ and p.

(e) (5 points) Solve for the demand (x_1, x_2) using only p, e_1, e_2.

(f) (5 points) Prove that x_2 is decreasing in p, so the demand is downward sloping.

3. Consider an economy with two goods, 1 and 2, and two agents, A and B. The utility function of each agent is

$$\begin{aligned} u_A(x_1, x_2) &= x_1 - \frac{8}{x_2}, \\ u_B(x_1, x_2) &= x_1 + 10 \log x_2. \end{aligned}$$

Suppose that the initial endowments are $(e_1^A, e_2^A) = (4, 4)$ and $(e_1^B, e_2^B) = (10, 3)$.

(a) (10 points) Consider the problem of maximizing the sum of utilities,

$$\begin{array}{ll} \text{maximize} & u_A(x_1^A, x_2^A) + u_B(x_1^B, x_2^B) \\ \text{subject to} & x_1^A + x_1^B \leq e_1^A + e_1^B, \\ & x_2^A + x_2^B \leq e_2^A + e_2^B. \end{array}$$

Let $(x_1^A, x_2^A, x_1^B, x_2^B)$ be a solution. Compute x_2^A and x_2^B.

(b) (10 points) Let the price be $p_1 = 1$ and $p_2 = p$. Compute the equilibrium price p and the allocation.

4. Consider an economy with I agents and L goods. Let $u_i(x)$ be the utility function of agent i, assumed to be locally nonsatiated. Let $p = (p_1, \ldots, p_L)$ be the price vector.

   (a) (5 points) What is the definition of local nonsatiation? You can explain in words.

   (b) (10 points) When solving for the equilibrium of an economy with locally nonsatiated utility functions, if $L - 1$ markets clear, the other market automatically clears. Explain in detail why this is true.

5. Consider an economy with two goods, 1 and 2, and two agents, A and B. The utility functions are

$$u_A(x_1, x_2) = 3 \log x_1 + \log x_2,$$
$$u_B(x_1, x_2) = \min \{x_1, x_2\}.$$

The endowments are $(e_1^A, e_2^A) = (8, 0)$ and $(e_1^B, e_2^B) = (4, 7)$.

(a) (2 points) What is the name of the type of agent A's utility function?

(b) (3 points) What is the name of the type of agent B's utility function?

(c) (5 points) Let the prices be $p_1 = 1$ and $p_2 = p$. Compute the demand of agent A.

(d) (5 points) Compute the demand (x_1^B, x_2^B) of agent B, using p.

(e) (5 points) Show that x_2^B is increasing in p.

(f) (5 points) Compute the competitive equilibrium.

You can detach this sheet and use as a scratch paper.