

# Econ 113 Mathematical Economics

## Midterm Exam 2

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November 12, 2015

Name: \_\_\_\_\_

Instruction:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, *i.e.*, base  $e = 2.718281828\dots$
- "Show" is synonymous to "prove".
- Full credit will not be given to correct but unsupported claims. Example:  $x^2 - 2x + 1 \geq 0$  is true but not obvious. You need to argue  $x^2 - 2x + 1 = (x - 1)^2 \geq 0$ .
- The exam time is 80 minutes.
- This exam has 4 questions on 6 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					



are

$$u_A(x_1, x_2, x_3) = \frac{1}{2} \log x_1 + \frac{1}{4} \log x_2 + \frac{1}{4} \log x_3,$$
$$u_B(x_1, x_2, x_3) = \frac{1}{3} \log x_1 + \frac{1}{3} \log x_2 + \frac{1}{3} \log x_3.$$

Each country can produce the consumption goods from labor using the linear technology  $y = a_{il}e$ , where  $e$  is labor input,  $y$  is output of good  $l$ , and  $a_{il} > 0$  is the productivity. Assume that productivities are

$$(a_{A1}, a_{A2}, a_{A3}) = (4, 2, 2),$$
$$(a_{B1}, a_{B2}, a_{B3}) = (1, 1, 2).$$

- (a) (5 points) What is the definition of comparative advantage of country  $A$  over  $B$ ? Compute the comparative advantage for each industry.
- (b) (5 points) Given the price  $p = (p_1, p_2, p_3)$  and the wage  $w_A$  of country  $A$ , compute the demand of country  $A$ .
- (c) (5 points) Assuming that both countries produce good 2 in free trade and setting  $p_2 = 1$ , compute  $p_1, p_3, w_A, w_B$ .
- (d) (5 points) Compute the free trade equilibrium consumption in each country.

(e) (5 points) Compute the labor allocation across each industry for each country.

3. Consider a world with  $L$  goods indexed by  $l = 1, \dots, L$ . Let  $p = (p_1, \dots, p_L)$  be the vector of world prices. Suppose that a small country (hence it does not affect world price  $p$ ) has  $I$  citizens indexed by  $i = 1, \dots, I$ , and let  $u_i(x)$  be the (locally nonsatiated) utility function of agent  $i$  and  $e_i$  be the initial endowment.

(a) (10 points) Suppose that the government is adopting some trade policy (tariff, quotas, etc.), and the equilibrium allocation is  $(x_i)$ . If the government is neither running a trade surplus nor a deficit, show that it must be

$$\sum_{i=1}^I p \cdot (e_i - x_i) = 0.$$

(b) (10 points) Find a trade policy that weakly Pareto improves the initial trade policy. Explain why your policy is weakly Pareto improving.

(c) (5 points) Currently Japan imposes a tariff of 400 Yen per kilogram on rice import (which is essentially the domestic rice price, so even if you import rice for free you will lose money) in order to protect the domestic rice producers. If you are an economist advising the Japanese government, what would you recommend to do, given the result of the previous question? (There is no “right” answer to this question since I am not giving all the details. Try to answer the best you can.)

4. Consider an economy with two countries,  $i = A, B$ , and two physical goods,  $l = 1, 2$ .



