Econ 113 Mathematical Economics Midterm Exam 2

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November 12, 2015

Name: ____

Instruction:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, *i.e.*, base e = 2.718281828...
- "Show" is synonymous to "prove".
- Full credit will not be given to correct but unsupported claims. Example: $x^2 2x + 1 \ge 0$ is true but not obvious. You need to argue $x^2 2x + 1 = (x 1)^2 \ge 0$.
- The exam time is 80 minutes.
- This exam has 4 questions on 6 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

- 1. Consider an economy with I agents with utility functions (u_i) . An allocation is denoted by (x_i) , where x_i is the consumption bundle of agent i.
 - (a) (5 points) What does it mean that an allocation (y_i) Pareto dominates the allocation (x_i) ? You can explain in words (maximum 4 points) or write down the precise mathematical definition.

(b) (5 points) Let the initial endowment be (e_i) . What does it mean that the allocation (x_i) is Pareto efficient? You can explain in words (maximum 4 points) or write down the precise mathematical definition.

(c) (5 points) What does the first welfare theorem say? You can explain in words.

(d) (10 points) How are the assumptions of the first and second welfare theorems different? You can explain in words.

2. Consider an economy with two countries, i = A, B, and three consumption goods, l = 1, 2, 3. Both countries have labor endowment $e_A = e_B = 6$. The utility functions

are

$$u_A(x_1, x_2, x_3) = \frac{1}{2} \log x_1 + \frac{1}{4} \log x_2 + \frac{1}{4} \log x_3,$$

$$u_B(x_1, x_2, x_3) = \frac{1}{3} \log x_1 + \frac{1}{3} \log x_2 + \frac{1}{3} \log x_3.$$

Each country can produce the consumption goods from labor using the linear technology $y = a_{il}e$, where e is labor input, y is output of good l, and $a_{il} > 0$ is the productivity. Assume that productivities are

$$(a_{A1}, a_{A2}, a_{A3}) = (4, 2, 2),$$

 $(a_{B1}, a_{B2}, a_{B3}) = (1, 1, 2).$

(a) (5 points) What is the definition of comparative advantage of country A over B? Compute the comparative advantage for each industry.

(b) (5 points) Given the price $p = (p_1, p_2, p_3)$ and the wage w_A of country A, compute the demand of country A.

(c) (5 points) Assuming that both countries produce good 2 in free trade and setting $p_2 = 1$, compute p_1, p_3, w_A, w_B .

(d) (5 points) Compute the free trade equilibrium consumption in each country.

(e) (5 points) Compute the labor allocation across each industry for each country.

- 3. Consider a world with L goods indexed by $l = 1, \ldots, L$. Let $p = (p_1, \ldots, p_L)$ be the vector of world prices. Suppose that a small country (hence it does not affect world price p) has I citizens indexed by $i = 1, \ldots, I$, and let $u_i(x)$ be the (locally nonsatiated) utility function of agent i and e_i be the initial endowment.
 - (a) (10 points) Suppose that the government is adopting some trade policy (tariff, quotas, etc.), and the equilibrium allocation is (x_i) . If the government is neither running a trade surplus nor a deficit, show that it must be

$$\sum_{i=1}^{I} p \cdot (e_i - x_i) = 0.$$

(b) (10 points) Find a trade policy that weakly Pareto improves the initial trade policy. Explain why your policy is weakly Pareto improving.

(c) (5 points) Currently Japan imposes a tariff of 400 Yen per kilogram on rice import (which is essentially the domestic rice price, so even if you import rice for free you will lose money) in order to protect the domestic rice producers. If you are an economist advising the Japanese government, what would you recommend to do, given the result of the previous question? (There is no "right" answer to this question since I am not giving all the details. Try to answer the best you can.)

4. Consider an economy with two countries, i = A, B, and two physical goods, l = 1, 2.

The endowment is $e_A = (10, 1)$ and $e_B = (1, 10)$. The utility function is

$$u(x_1, x_2) = \frac{1}{2}\log x_1 + \frac{1}{2}\log x_2$$

for all agents. Suppose that there are transportation costs, and 50% of the exported goods perish by the time they reach the destination.

- (a) (5 points) How many kinds of goods are there in the world? Answer the number and explain the reason.
- (b) (5 points) Explain why a model of international trade with transportation costs can be regarded as a standard Arrow-Debreu model.

(c) (5 points) Assuming that country A imports good 2, what is its price? (Set the price of good 1 equal to 1.)

(d) (10 points) Compute the free trade equilibrium. Make sure to compute all prices, consumption, and import/exports in each country.