Mathematical Economics
Midterm Exam

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Name: _____

Instruction:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, *i.e.*, base e $= 2.718281828\ldots$.
- "Show" is synonymous to "prove".
- Full credit will not be given to correct but unsupported claims. Example: $x^2 - 2x + 1 \geq 0$ is true but not obvious. You need to argue $x^2 - 2x + 1 = (x-1)^2 \geq 0$.
- The exam time is 80 minutes.
- This exam has 4 questions on 8 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	25	20	30	25	100
Score:					

1. Consider an economy with L goods and I agents. Agent i has endowment e_i and a locally nonsatiated utility function $u_i(x)$. Let p be the price vector. If the notation bothers you, you may set $L = 2$ and $I = 2$.

   (a) (5 points) What is the definition of local nonsatiation? You may explain in words (maximum 4 points) or mathematically.

   (b) (5 points) Suppose x_i solves the utility maximization problem

   $$\text{maximize } u_i(x) \text{ subject to } p \cdot x \leq p \cdot e_i.$$

   Explain why it must be the case that $p \cdot x_i = p \cdot e_i$.

   (c) (5 points) What does it mean that an allocation (y_i) Pareto dominates the allocation (x_i)? You can explain in words (maximum 4 points) or write down the precise mathematical definition.

   (d) (3 points) What does it mean that the feasible allocation (x_i) is Pareto efficient? You can explain in words.

   (e) (7 points) Let $\{p, (x_i)\}$ be an Arrow-Debreu equilibrium. Prove that (x_i) is Pareto efficient.

2. Consider an economy with two agents indexed by $i = 1, 2$ and two goods indexed by $l = 1, 2$. The utility functions are

$$u_1(x_1, x_2) = -\frac{1}{x_1} - \frac{1}{x_2},$$
$$u_2(x_1, x_2) = \frac{2}{3}\log x_1 + \frac{1}{3}\log x_2,$$

and the initial endowments are $e_1 = e_2 = (1, 6)$.

(a) (8 points) Is the initial endowment Pareto efficient? Answer yes or no, then explain why.

(b) (6 points) Compute the Pareto efficient allocation in which agent 1 consumes 1 unit of good 1.

(c) (6 points) Compute the competitive equilibrium with transfer payments when the allocation is the one in the previous question. (Normalize the price of good 1 to be 1, so $p_1 = 1$.)

3. Consider an economy with two goods indexed by $l = 1, 2$. Suppose that there is a small country (so it doesn't affect world prices) with two agents indexed by $i = 1, 2$ and endowments $e_1 = (3/2, 1)$, $e_2 = (1, 3/2)$. All agents have utility function

$$u(x_1, x_2) = x_1 x_2.$$

Below, always normalize the price of good 1 to be $p_1 = 1$.

(a) (5 points) Compute the autarky equilibrium allocation and price.

(b) (7 points) Suppose that the country opens up to trade, and the price of good 2 changes to $p_2 = 1/2$. Compute the free trade allocation and utility and determine who gains/loses from trade.

(c) (7 points) Suppose that the government imposes a tariff of $\tau = 1/4$ on the import of good 2, and the domestic price of good 2 becomes $q = p_2 + \tau = 1/2 + 1/4 = 3/4$. Let T be the tax revenue from the tariff, and suppose that the government gives out the tariff revenue equally to agents (so each agent gets $T/2$). Derive an equation that T satisfies.

(d) (6 points) Solve for the new allocation and show that all agents gain from trade.

(e) (5 points) Propose a better policy than the government's. Compute the allocation and utility under your suggested policy.

4. Consider an economy with two countries, $i = A, B$, and two physical goods, $l = 1, 2$. The endowment is $e_A = (9, 2)$ and $e_B = (2, 9)$. The utility functions are

$$u_A(x_1, x_2) = \frac{2}{3} \log x_1 + \frac{1}{3} \log x_2,$$
$$u_B(x_1, x_2) = \frac{1}{3} \log x_1 + \frac{2}{3} \log x_2.$$

Suppose that there are transportation costs, and one third (1/3) of the exported goods perish by the time they reach the destination.

(a) (5 points) How many kinds of goods are there in the world? Answer the number and explain the reason.

(b) (5 points) Explain why a model of international trade with transportation costs can be regarded as a standard Arrow-Debreu model.

(c) (5 points) Assuming that country A imports good 2, what is its price? (Set the price of good 1 equal to 1.)

(d) (10 points) Compute the free trade equilibrium. Make sure to compute all prices, consumption, and import/exports in each country.

You can detach this sheet and use as a scratch paper.