Mathematical Economics
Midterm Exam

Prof. Alexis Akira Toda

November 16, 2017

Name: _____

Instruction:

- Read these instructions and the questions carefully.

- Don't start the exam until instructed.

- Turn off any electronic devices and put them in your bag.

- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.

- All logarithms are natural logarithms, *i.e.*, base e = 2.718281828 . . . .

- "Show" is synonymous to "prove".

- Full credit will not be given to correct but unsupported claims. Example: $x^2 - 2x + 1 \geq 0$ is true but not obvious. You need to argue $x^2 - 2x + 1 = (x-1)^2 \geq 0$.

- The exam time is 80 minutes.

- This exam has 4 questions on 8 pages excluding the cover page, for a total of 130 points.

- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.

- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	25	35	35	35	130
Score:					

1. Consider an economy with I agents with utility functions (u_i). An *allocation* is denoted by (x_i), where x_i is the consumption bundle of agent i.

   (a) (5 points) What does it mean that an allocation (y_i) Pareto dominates the allocation (x_i)? You can explain in words (maximum 4 points) or write down the precise mathematical definition.

   (b) (5 points) Let the initial endowment be (e_i). What does it mean that the allocation (x_i) is Pareto efficient? You can explain in words (maximum 4 points) or write down the precise mathematical definition.

   (c) (5 points) What does the first welfare theorem say? You can explain in words.

   (d) (10 points) How are the assumptions of the first and second welfare theorems different? You can explain in words.

2. Consider an economy with two countries $(i = 1, 2)$ and two goods $(l = 1, 2)$. Each country consists of a single agent type whose utility function is

$$u(x_1, x_2) = x_1 x_2.$$

Suppose that the labor endowments are $e_A = 10$, $e_B = 1$, and the vector of labor productivities are

$$(a_{A1}, a_{A2}) = (12, 6)$$
$$(a_{B1}, a_{B2}) = (4, 4).$$

(a) (10 points) Compute the autarky equilibrium in country A and the utility level. Note that you need to compute prices, wage, and allocations of goods and labor. Normalize the price of good 1 so that $p_1 = 1$.

(b) (5 points) Repeat the previous question for country B.

(c) (10 points) Compute the free trade equilibrium.

(d) (10 points) Explain why during international conflicts, large/developed countries often try to impose an embargo on small/developing countries. (Imagine the U.S.-Japan relationship before WWII or U.S.-North Korea now.)

3. Consider an economy with three agents ($i = 1, 2, 3$), two goods ($l = 1, 2$), and two countries, A, B. Agents 1 and 2 live in country A and agent 3 lives in country B. The utility functions are

$$u_1(x_1, x_2) = x_1^2 x_2,$$
$$u_2(x_1, x_2) = x_1 x_2^2,$$
$$u_3(x_1, x_2) = x_1 x_2.$$

Suppose that the initial endowments are $e_1 = e_2 = (3, 3)$ and $e_3 = (18, 6)$. In answering questions below, in order to make the notation consistent use x_{il} for consumption of good l by agent i. (So x_{12} is consumption of good 2 by agent 1, for example.) Also, use $p_1 = 1$ and $p_2 = p$ for the prices.

(a) (5 points) Compute the competitive equilibrium when country A is in autarky as well as the utility level of each agent.

(b) (10 points) Compute the free trade equilibrium price and allocation.

(c) (5 points) Compute the utility level of each agent and determine who gained from trade and who lost.

(d) (15 points) Find a tax scheme in country A such that free trade is Pareto improving. Explain why the tax scheme you suggest is Pareto improving.

4. Consider Ricardo's international trade model with two countries, $i = A, B$, and L consumption goods indexed by $l = 1, \ldots, L$. Let $a_{il} > 0$ be the labor productivity of country i when producing good l. Assume that the values $\{a_{Al}/a_{Bl}\}_{l=1}^{L}$ are all distinct.

(a) (5 points) Define the notion of comparative advantage.

(b) (15 points) Let $w_i > 0$ be the wage rate in country i and $p_l > 0$ be the price of good l. In equilibrium, prove that $p_l a_{il} \leq w_i$, with equality if country i produces good l.

(c) (15 points) Let $L = \{1, \ldots, L\}$ be the set of goods. Let L_i $(i = A, B)$ be the set of goods produced by country i in equilibrium. Prove that $L_A \cup L_B = L$ and $L_A \cap L_B$ is either empty or consists of a single element.

You can detach this sheet and use as a scratch paper.