

Econ 113 Mathematical Economics

Midterm Exam

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Name: _____

Instruction:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, i.e., base $e = 2.718281828\dots$
- "Show" is synonymous to "prove".
- Full credit will not be given to correct but unsupported claims. Example: $x^2 - 2x + 1 \geq 0$ is true but not obvious. You need to argue $x^2 - 2x + 1 = (x - 1)^2 \geq 0$.
- The exam time is 80 minutes.
- This exam has 4 questions on 7 pages excluding the cover page, for a total of 120 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	30	30	30	30	120
Score:					

1. Consider an economy with two physical goods (apples and bananas) and two periods denoted by $t = 1, 2$. There is only one agent (type) with utility

$$U(x_1, y_1, x_2, y_2) = u(x_1, y_1) + \beta u(x_2, y_2),$$

where

$$u(x, y) = \alpha \log x + (1 - \alpha) \log y$$

is the period utility function, $\beta > 0$ is the discount factor, and x_t, y_t denote the consumption of apples and bananas in period t . Let the endowment of apples and bananas in period t be a_t and b_t .

- (a) (10 points) Compute the gross real interest rate in units of apples.
- (b) (10 points) Compute the gross real interest rate in units of bananas.
- (c) (10 points) Under what condition do the two interest rates coincide? Is it surprising that the two rates are distinct except a very special case?

2. (a) (10 points) What is the definition of local nonsatiation? Provide both intuitive and formal definitions.

(b) (10 points) What is the definition of Pareto efficiency?

(c) (10 points) First Welfare Theorem says that if utility functions are locally non-satiated, then a competitive equilibrium allocation is Pareto efficient. Construct an example such that utility functions are not necessarily locally nonsatiated and the competitive equilibrium is Pareto inefficient.

3. Consider an economy with two countries ($i = 1, 2$) and two goods ($l = 1, 2$). Each

country consists of a single agent type whose utility function is

$$u(x_1, x_2) = x_1 x_2.$$

Suppose that the labor endowments are $e_A = 1$, $e_B = e$, and the vector of labor productivities are

$$\begin{aligned}(a_{A1}, a_{A2}) &= (2, 1) \\ (a_{B1}, a_{B2}) &= (1, 1).\end{aligned}$$

Below, normalize the price of good 1 so that $p_1 = 1$.

- (a) (5 points) Define the notion of comparative advantage.
- (b) (10 points) Under what condition on e is there a free trade equilibrium in which country A produces both goods? Compute the equilibrium.
- (c) (5 points) Under what condition on e is there a free trade equilibrium in which country B produces both goods? Compute the equilibrium.

(d) (10 points) Under what condition on e is there a free trade equilibrium in which each country produces only one good? Compute the equilibrium.

4. (30 points) Suppose you are an economist advising a president of some country. Suppose the president would like to impose tariffs against China. Convince the president that that is a bad policy. Assume the president is intelligent and would not be convinced unless you demonstrate using formal mathematical reasoning.

You can detach this sheet and use as a scratch paper.