ECONOMICS 113:
Mathematical Economics

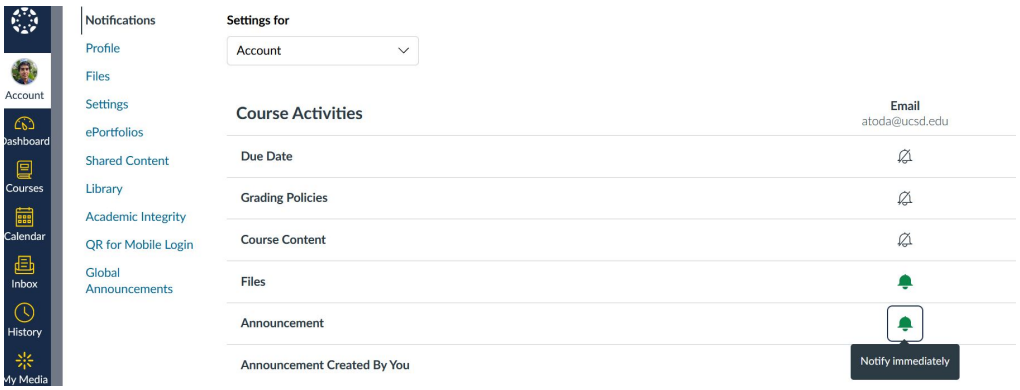
Fall 2023

Basic information

Lectures	Tu/Th 14:00-15:20, SOLIS 105
Instructor	Prof. Alexis Akira Toda
Office hours	TBA
Email	atoda@ucsd.edu
Webpage	https://alexisakira.github.io/
	(Go to `Teaching` → `Mathematical Economics`)
TA	Connor Goldstick, cgoldsti@ucsd.edu

Canvas

I will be using Canvas (https://canvas.ucsd.edu) to communicate with you. Please access it and familiarize yourself. We do not monitor Canvas Inbox; please do not use it. Most importantly, please turn on the notification setting to receive announcement notifications. Go to `Account` → `Notifications` and select "Notify immediately" for "Announcement". See the screen shot below. Make sure to also check your spam folders. In the past, I received complaints such as "we didn't receive announcements" but this is an issue at the student's end.



Course description

Carl Friedrich Gauss said mathematics is the queen of the sciences (and number theory is the queen of mathematics).[1] Paul Samuelson said economics is the queen of the social

[1] http://en.wikiquote.org/wiki/Carl_Friedrich_Gauss

sciences.[2] Not surprisingly, modern economics is a highly mathematical subject.

Mathematical economics studies the mathematical foundations of economic theory in the approach known as the Arrow-Debreu model of general equilibrium. Partial equilibrium (things like demand and supply curves), which you have probably learned in Econ 100ABC, considers each market separately. General equilibrium (GE for short), on the other hand, considers the economy as a whole, taking into account the interaction of all markets.

Econ 113 is probably the most mathematically advanced undergraduate course offered at UCSD Economics Department, but it should have a high return. In the course, we will develop a mathematical model of classical economic thoughts like Bentham's "greatest happiness principle" and Smith's "invisible hand", and prove theorems. Then we will apply the theory to international trade, finance, social security, etc. Time permitting, I will talk about my own research.

Prerequisites

Calculus, linear algebra, upper division microeconomic theory (at UCSD these courses are Math 20ABC, Math 18, and Econ 100ABC) are required. Students with strong mathematical background (typically including one quarter of real analysis, UCSD Math 140A or 142A) may enroll without economics prerequisites. **Although not required, because the lectures are "proof-based", prior or concurrent enrollment in Math 109 or Math 140A is highly recommended.** Students with inadequate mathematical background should take other mathematics courses before enrolling in Econ 113.

Course material

There are three important components of the course material, which are

- Lecture notes,
- Slides,
- Question bank.

These are all posted at my personal website at https://alexisakira.github.io/teaching/mathematical-economics and will be updated regularly.

The lectures will be based on on my lecture notes, and I will mostly use slides to explain the essential points. However, I expect students to actually read the lecture notes to reinforce understanding. The question bank is optional but I highly recommend you to study some of it, as it is essentially the universe of my past exam questions (undergraduate mathematical economics, graduate core microeconomics, and graduate qualifying exam).

The following textbooks may be useful, though none of them are required.

Debreu (1959) Classic, concise description of the general equilibrium theory.

Mas-Colell et al. (1995) Standard graduate-level textbook.

Starr (2011) This book is written by UCSD Emeritus Professor Ross Starr, who has taught this course for many years before I took over. It does not contain many examples but is self-contained and pedagogic in that it proceeds from the easy

[2] Samuelson "Economics", 10th edition, preface.

and special case to the difficult and general case. An added bonus is an accessible proof of the Brouwer fixed point theorem.

Bewley (2007) This book is roughly at the same level as Starr (2011), but contains lots of examples and exercises and thus may complement your study.

The course will use a lot of math, as the course title suggests. Relevant topics are basic linear algebra (but not much), calculus, convex analysis, and constrained optimization. For the last two topics, my EME book draft[3] is more than sufficient, but I will cover them briefly in class so you don't need to study in advance.

Preliminary course outline

1. Introduction
2. Definition of Arrow-Debreu model
3. Crash course in convex analysis and convex programming
4. Quasi-linear model: mathematical formulation of Bentham's "greatest happiness principle"
5. First and second welfare theorems: mathematical formulation of Smith's "invisible hand"
6. Existence of equilibrium (correspondences, maximum theorem, Brouwer and Kakutani fixed point theorems)
7. Computation and uniqueness of equilibrium
8. International trade
9. Finance

Evaluation

Evaluation will be based on problem sets (50%) and a final exam (50%).

Problem sets I will assign a problem set every two weeks, in total 5 or 6 problem sets. You need to turn in a scanned copy of **legible and hand-written** solution through gradescope. Each problem set will be graded on a 0-10 scale, but the possible points will be an element of the set $\{0, 5, 8, 10\}$ according to the following criteria.

- 0 points if you do not turn in the problem set.
- 5 points if you turn in but the analysis is far from correct or complete.
- 10 points if you turn in and the analysis is mostly correct or complete.
- 8 points if you turn in and the analysis is between compete and incomplete.

We understand that the distinction between 5, 8, 10 could be ambiguous but our decision is final. However, to alleviate your stress and concerns, we will drop the lowest score from the final grade. This means that if there are 6 problem sets, only your 5 best scores will count towards the final grade.

[3] https://alexisakira.github.io/files/EME.pdf

Final exam The final exam will be an oral exam (about 10 minutes per student) implemented on Zoom. According to my experience, although oral exams are time consuming to implement, it is close to ideal because I can tell whether you understand the material, you will not be penalized for algebraic mistakes, and academic integrity is maintained.

Questions

The best opportunity to ask questions is *during* the class, for two reasons. First, you can resolve your question immediately (assuming—well—I know the answer). Second, your classmates are likely to have similar questions, so they can benefit from questions being resolved and I benefit by saving time. So, don't be shy, please ask questions.

How to do well in this course

Get your favorite math text (linear algebra and calculus) or my lecture notes of Econ 205 at hand so that you can refer if necessary. Experience tells that (this is *true*) students who regularly attend classes outperform those who don't, so come to class. Ask questions during the class whenever you don't understand. Read the lecture notes. Solve question bank problems. If you do (very) well in this course, you have a good chance to be admitted to good Master (PhD) programs (and therefore get a lucrative job in the future).

Miscellaneous

- Email writing tips:
  https://alexisakira.github.io/misc/email
- Letter of recommendation policy:
  https://alexisakira.github.io/misc/letter-of-recommendation

References

Truman F. Bewley. *General Equilibriuim, Overlapping Generations Models, and Optimal Growth Theory*. Harvard University Press, Cambridge, MA, 2007.

Gerard Debreu. *Theory of Value*. Cowles Foundation Monograph 17. Yale University Press, New Haven, 1959.

Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green. *Microeconomic Theory*. Oxford University Press, New York, 1995.

Ross M. Starr. *General Equilibrium Theory: An Introduction*. Cambridge University Press, 2 edition, 2011.