Econ 200B General Equilibrium
Midterm Exam

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Name: ______________________________

Instructions:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, *i.e.*, base $\mathrm{e} = 2.718281828\dots$.
- "Show" or "prove" require a formal mathematical proof or argument. "Explain" only requires an intuitive, though logically coherent, verbal argument.
- This exam has 5 questions on 10 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	5	Total
Points:	15	15	15	15	40	100
Score:						

1. Consider an economy with two goods, 1 and 2, and two agents, A and B. The utility function of each agent is

$$u_A(x_1, x_2) = x_1 - \frac{8}{x_2},$$
$$u_B(x_1, x_2) = x_1 + 10 \log x_2.$$

Suppose that the initial endowments are $(e_1^A, e_2^A) = (4, 4)$ and $(e_1^B, e_2^B) = (10, 3)$.

(a) (2 points) What is the name of these utility functions?

(b) (8 points) Consider the problem of maximizing the sum of utilities,

$$\begin{array}{ll} \text{maximize} & u_A(x_1^A, x_2^A) + u_B(x_1^B, x_2^B) \\ \text{subject to} & x_1^A + x_1^B \leq e_1^A + e_1^B, \\ & x_2^A + x_2^B \leq e_2^A + e_2^B. \end{array}$$

Let $(x_1^A, x_2^A, x_1^B, x_2^B)$ be a solution. Compute x_2^A and x_2^B.

(c) (5 points) Let the price be $p_1 = 1$ and $p_2 = p$. Compute the equilibrium price p and the allocation.

2. Let $\mathcal{E} = \{I, (e_i), (u_i)\}$ be an Arrow-Debreu economy with I agents and L goods.

   (a) (5 points) What is the first welfare theorem? State the assumptions as well as the conclusion.

   (b) (10 points) Prove the first welfare theorem.

3. Consider an economy with I agents and L basic goods labeled by $l = 1, \ldots, L$. Suppose that there is another good, labeled 0, which is a public good. (A public good is non-excludable, *i.e.*, the consumption of one agent does not reduce the availability of that good to other agents. Therefore all agents consume the same amount of good 0, which equals aggregate supply in equilibrium.) Suppose that there are no endowments of good 0, which is produced from other goods $l = 1, \ldots, L$ using some technology represented by a production function $y = f(x_1, \ldots, x_L)$. It is well known that the presence of a public good may make the economy inefficient. Let $u_i(x_0, x_1, \ldots, x_L)$ be the utility function of agent i, assumed to be locally nonsatiated.

   (a) (7 points) Show that by quoting an individual-specific price for the public good, we can make the competitive equilibrium allocation efficient. (Hint: expand the set of goods, and consider an economy with $L + I$ goods labeled by $l =$

$1, \ldots, L+I$. Goods $l = 1, \ldots, L$ are the L basic goods, and good $L+i$ is the public good consumed by agent i. Make sure to discuss how we should reinterpret the production technology and market clearing conditions.)

(b) (8 points) Suppose that there are two agents ($i = 1, 2$), one basic good, and a public good. Agent i has utility function

$$u_i(x_0, x_1) = \alpha_i \log x_0 + (1 - \alpha_i) \log x_1,$$

where x_0, x_1 are consumption of the public good and the basic good. Let e_i be the initial endowment of agent i's basic good, and suppose there is a technology that converts the basic good to the public good one-for-one. Normalize the price of the basic good to be 1. Find individual-specific prices for the public good to make the competitive equilibrium allocation efficient.

4. Consider an economy with three agents ($i = 1, 2, 3$), two goods ($l = 1, 2$), and two countries, A, B. Agents 1 and 2 live in country A and agent 3 lives in country B. All agents have utility function

$$u(x_1, x_2) = x_1 x_2.$$

Suppose that the initial endowments are $e_1 = (16, 1)$, $e_2 = (4, 4)$, and $e_3 = (10, 25)$. Below, always normalize the price of good 1 to be 1.

(a) (5 points) Compute the equilibrium price, allocation, and utility levels when country A is in autarky.

(b) (5 points) Compute the free trade equilibrium price, allocation, and utility levels of agents 1 and 2.

(c) (5 points) Concerned with agent 2's welfare, suppose that the government of country A introduces a direct tax/subsidy to its citizens. Find a tax scheme in country A that makes the free trade allocation weakly Pareto dominant to the autarky allocation.

5. Consider an economy with two agents ($i = 1, 2$), two periods ($t = 0, 1$), and two states ($s = u, d$) at $t = 1$. The aggregate endowment is $e = (e_0, e_u, e_d)$, where $e_0 > 0$ and $e_u > e_d > 0$ (so u, d are the "up" and "down" states). The initial endowments are $e_1 = \alpha e$ and $e_2 = (1-\alpha)e$, so the wealth share of agent 1 is $0 < \alpha < 1$. Agent 1's subjective probability of state s is $\pi_s > 0$, where $\pi_u + \pi_d = 1$. The utility functions are

$$
\begin{aligned}
u_1(x_0, x_u, x_d) &= (1-\beta)\log x_0 + \beta(\pi_u \log x_u + \pi_d \log x_d), \\
u_2(x_0, x_u, x_d) &= (1-\beta)\log x_0 + \beta \log(\min\{x_u, x_d\}),
\end{aligned}
$$

where $0 < \beta < 1$ is the discount factor. Let $p = (1, p_u, p_d)$ be the price vector and $W = e_0 + p_u e_u + p_d e_d$ be the value of aggregate endowment. Note that this is an Arrow-Debreu economy, so all trades occur at $t = 0$.

(a) (5 points) Explain why there exists a unique equilibrium.

(b) (3 points) Compute the demand of agent 1.

(c) (3 points) Compute the demand of agent 2.

(d) (4 points) Let a "stock" be an asset that pays out the aggregate endowment at $t = 1$. Let a "risk-free asset" be an asset that pays out 1 at $t = 1$ no matter what. Let q_s, q_f be the price of the stock and risk-free asset at $t = 0$. Express q_s, q_f using only p_u, p_d, e_u, e_d.

(e) (4 points) Express p_u, p_d using only q_s, q_f, e_u, e_d.

(f) (5 points) Using the market clearing condition for $t = 0$, compute q_s using only exogenous parameters.

(g) (6 points) Using the market clearing condition for $s = u, d$, show that

$$\frac{\alpha\beta}{1-\beta} e_0 \left(\frac{\pi_u}{p_u} - \frac{\pi_d}{p_d} \right) = e_u - e_d.$$

(h) (5 points) Show that the risk-free rate goes up when agent 1 becomes relatively richer (*i.e.*, α gets larger).

(i) (5 points) Show that the risk-free rate goes up when agent 1 becomes more optimistic (*i.e.*, π_u gets larger).

You can detach this sheet and use as a scratch paper.