Econ 200B General Equilibrium
Midterm Exam

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Name: ______________________________

Instructions:

- Read these instructions and the questions carefully.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, *i.e.*, base $\mathrm{e} = 2.718281828\dots$.
- "Show" or "prove" require a formal mathematical proof or argument. "Explain" only requires an intuitive, though logically coherent, verbal argument.
- This exam has 5 questions on 12 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1. Consider an economy consisting of two agents, $i = A, B$, and two goods, $l = 1, 2$. The initial endowments are $e_A = a = (a_1, a_2) \gg 0$, $e_B = b = (b_1, b_2) \gg 0$, and agents have Cobb-Douglas utility functions

$$U_A(x_1, x_2) = \alpha \log x_1 + (1 - \alpha) \log x_2,$$
$$U_B(x_1, x_2) = \beta \log x_1 + (1 - \beta) \log x_2,$$

where $\alpha, \beta \in (0, 1)$. Below, normalize the price of good 2 to be $p_2 = 1$ and set $p_1 = p$.

(a) (3 points) Let $w = pa_1 + a_2$ be the wealth of agent A. Write down the Lagrangian of the utility maximization problem.

(b) (4 points) Derive the first-order conditions.

(c) (4 points) Express the demand of agent A using only α, a_1, a_2, p.

(d) (4 points) Compute the equilibrium price.

(e) (5 points) Show that if either (i) the supply of good 1 decreases (a_1 or b_1 goes down), (ii) the supply of good 2 increases (a_2 or b_2 goes up), or (iii) agents like good 1 more (α or β goes up), then the price of good 1 increases.

2. Let $\mathcal{E} = \{I, (e_i), (u_i)\}$ be an Arrow-Debreu economy. Suppose that for all i we have $e_i \gg 0$ and the utility function $u_i : \mathbb{R}^L_+ \to \mathbb{R}$ is continuous, quasi-concave, and locally nonsatiated. Suppose in addition that for some i, u_i is weakly monotonic.

   (a) (3 points) What is the definition of local nonsatiation?

   (b) (3 points) What is the definition of weak monotonicity?

   (c) (14 points) Show that $\mathcal{E}$ has a competitive equilibrium $\{p, (x_i)\}$ in which all markets clear exactly, that is,

$$\sum_{i=1}^{I} x_i = \sum_{i=1}^{I} e_i.$$

   You can use (without proofs) the existence results explained in the lecture note.

3. (a) (2 points) Let $f(x) = -\frac{1}{\gamma}\mathrm{e}^{-\gamma x}$, where $\gamma > 0$. Compute $-\frac{f''(x)}{f'(x)}$.

(b) (3 points) Consider an economy with two goods that can be consumed in any amounts (positive or negative), and suppose that an agent has endowment $e = (a, b)$ and an additively separable utility function

$$U(x, y) = u(x) + v(y),$$

where $u' > 0$, $u'' < 0$, and similarly for v. Let $p_1 = p$ and $p_2 = 1$ be prices. Show that the agent's demand (x, y) satisfies

$$u'(x) - pv'(pa + b - px) = 0.$$

(c) (3 points) Regard x, the demand for good 1, as a function of the price p. Show that

$$\frac{\partial x}{\partial p} = \frac{v'(y) + pv''(y)(a-x)}{u''(x) + p^2 v''(y)},$$

where $y = pa + b - px$.

(d) (3 points) Show that

$$\frac{\partial x}{\partial p} = -\frac{1 - p\gamma_v(y)(a-x)}{p\gamma_u(x) + p^2\gamma_v(y)},$$

where $\gamma_u(x) = -\frac{u''(x)}{u'(x)}$ and similarly for v.

(e) (3 points) Suppose that there are I agents indexed by $i = 1, \ldots, I$. Agent i's endowment is $e_i = (a_i, b_i)$ and the utility function is

$$U_i(x,y) = -\frac{1}{\gamma_i}\left(\alpha_i \mathrm{e}^{-\gamma_i x} + (1-\alpha_i)\mathrm{e}^{-\gamma_i y}\right),$$

where $\gamma_i > 0$ and $0 < \alpha_i < 1$. Let x_i be agent i's demand for good 1. Show that

$$\frac{\partial x_i}{\partial p} = -\frac{1}{\gamma_i p(1+p)} + \frac{1}{1+p}(a_i - x_i).$$

(f) (3 points) Let $z_1(p) = \sum_{i=1}^{I}(x_i - a_i)$ be the aggregate excess demand for good 1. Show that if p is an equilibrium price, then $z_1'(p) < 0$.

(g) (3 points) Show that the equilibrium is unique.

4. Consider an economy with two goods indexed by $l = 1, 2$. Suppose that there is a small country (so it doesn't affect world prices) with two agents indexed by $i = 1, 2$ and endowments $e_1 = (3/2, 1)$, $e_2 = (1, 3/2)$. All agents have utility function

$$u(x_1, x_2) = x_1 x_2.$$

Below, always normalize the price of good 1 to be $p_1 = 1$.

(a) (3 points) Compute the autarky equilibrium allocation and price.

(b) (4 points) Suppose that the country opens up to trade, and the price of good 2 changes to $p_2 = 1/2$. Compute the free trade allocation and utility and determine who gains/loses from trade.

(c) (4 points) Suppose that the government imposes a tariff of $\tau = 1/4$ on the import of good 2, and the domestic price of good 2 becomes $q = p_2 + \tau = 1/2 + 1/4 = 3/4$. Let T be the tax revenue from the tariff, and suppose that the government gives out the tariff revenue equally to agents (so each agent gets $T/2$). Express T using q, τ, T.

(d) (4 points) Solve for the new allocation and show that all agents gain from trade.

(e) (5 points) Propose a better policy than the government's. Compute the allocation and utility under your suggested policy.

5. Consider an economy with two periods, denoted by $t = 0, 1$, and three agents, denoted by $i = 1, 2, 3$. There are two states at $t = 1$, denoted by $s = 1, 2$. The two states

occur with equal probability $\pi_1 = \pi_2 = 1/2$. Suppose that agent i's utility function is

$$U_i(x_0, x_1, x_2) = u_i(x_0) + \pi_1 u_i(x_1) + \pi_2 u_i(x_2),$$

where x_0, x_1, x_2 denote the consumption at $t = 0$ and states $s = 1, 2$, and the Bernoulli utility functions $u_i(x)$ are given by

$$\begin{aligned} u_1(x) &= -\frac{4}{x}, \\ u_2(x) &= -\frac{4}{x-2}, \\ u_3(x) &= -\frac{4}{x+2}. \end{aligned}$$

The initial endowments $e_i = (e_{i0}, e_{i1}, e_{i2})$ are given by

$$\begin{aligned} e_1 &= (3, 5, 1), \\ e_2 &= (3, 3, 3), \\ e_3 &= (2, 4, 2). \end{aligned}$$

(a) (2 points) What is the name of this type of utility functions?

(b) (2 points) For a given level of consumption, which agent is the most risk averse? Answer based on reasoning.

(c) (6 points) Normalize the price of $t = 0$ good to be $p_0 = 1$. Compute the equilibrium state prices p_1, p_2.

(d) (3 points) Compute the (gross) risk-free interest rate.

(e) (3 points) Consider an asset (stock) that pays out the aggregate endowment as dividend. Compute the ex-dividend stock price (the stock price excluding the dividend) at $t = 0$.

(f) (2 points) Compute the expected stock return at $t = 0$ and show that it is higher than the risk-free rate.

(g) (2 points) Compute the price at $t = 0$ of a put option written on a stock with strike price 9.

You can detach this sheet and use as a scratch paper.