Econ 200B General Equilibrium
Midterm Exam

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Name: ____

Instructions:

- Read these instructions and the questions carefully. Questions are not necessarily ordered in the order of difficulty.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, i.e., base $\mathrm{e} = 2.718281828\dots$.
- "Show" or "prove" require a formal mathematical proof or argument. "Explain" only requires an intuitive, though logically coherent, verbal argument.
- This exam has 4 questions on 9 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	10	20	30	40	100
Score:					

1. (10 points) As discussed during lectures, the equilibrium of an Arrow-Debreu economy need not be unique. Discuss conditions under which the equilibrium is unique. (No proofs are required but the more exhaustive the list is, the better.)

2. (20 points) Consider an Arrow-Debreu economy with two agents denoted by $i = A, B$ and S states denoted by $s = 1, \ldots, S$. Let $\pi_s > 0$ be the objective probability of state s, where $\sum_{s=1}^{S} \pi_s = 1$. Let $e_{is} > 0$ be agent i's initial endowment of good s. Suppose that the utility functions are given by

$$U_A(x) = \sum_{s=1}^{S} \pi_s u(x_s),$$

$$U_B(x) = \sum_{s=1}^{S} \pi_s x_s,$$

where $u' > 0$, $u'' < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. What is the most you can say about equilibrium prices and allocations?

3. Consider an Arrow-Debreu economy with two agents indexed by $i = 1, 2$. Suppose that the utility functions are

$$U_1(x_1, x_2) = \alpha \log x_1 + (1 - \alpha) \log x_2,$$
$$U_2(x_1, x_2) = \min\{x_1, x_2\},$$

where $0 < \alpha < 1$ is a preference parameter. Let agent i's initial endowment be (e_{i1}, e_{i2}). Let $p_1 = 1$ and $p_2 = p$ be the prices.

(a) (10 points) Compute each agent's demand for good 1, given p.

(b) (10 points) Derive a necessary and sufficient condition such that p is an equilibrium price for some $e_{12} > 0$.

(c) (10 points) Show that when agent 1 likes good 1 more (α increases), the relative price of good 1, which is $1/p$, increases (so p decreases).

4. Consider the following general equilibrium model. There are three time periods indexed by $t = 0, 1, 2$. There is a continuum of ex ante identical agents, where the population is normalized to 1. At $t = 0$, agents are endowed with $e > 0$ units of consumption good. At $t = 0$, agents can invest goods in two technologies. One unit of investment in technology 1 yields 1 unit of good at $t = 1$. One unit of investment in technology 2 yields $R > 0$ units of good at $t = 2$. Agents get utility only from consumption at $t = 1, 2$. At the beginning of $t = 1$, agents get "liquidity shocks", and with probability $\pi_i > 0$, their utility function becomes

$$U_i(x_1, x_2) = (1 - \beta_i) \log x_1 + \beta_i \log x_2,$$

where $\beta_i \in (0, 1)$ is the discount factor of type i and $\sum_{i=1}^{I} \pi_i = 1$. Without loss of generality, assume

$$\beta_1 < \cdots < \beta_I,$$

so a type with a smaller index is more impatient. Suppose that the ex ante utility is

$$U((x_{i1}, x_{is})_i) = \sum_{i=1}^{I} \pi_i \alpha_i U_i(x_{i1}, x_{i2}),$$

where $\alpha_i > 0$ is the weight on type i such that

$$\alpha_1 > \cdots > \alpha_I,$$

so agents care about emergencies in the sense that they put more utility weight on the impatient type. Note that we assume the law of large numbers, so at $t = 1$, exactly fraction $\pi_i > 0$ of agents are of type i. After observing their patience type at $t = 1$, agents can trade consumption for $t = 1, 2$ at a competitive (Arrow-Debreu) market.

(a) (10 points) In general, let f, g be strictly increasing functions and X be a random variable. Prove the Chebyshev inequality

$$E[f(X)g(X)] \geq E[f(X)]E[g(X)],$$

with equality if and only if X is constant almost surely.

(Hint: let X' be an i.i.d. copy of X and consider the expectation of the quantity $(f(X) - f(X'))(g(X) - g(X')) \geq 0$.)

(b) (5 points) Noting that agents are ex ante identical, at $t = 0$ they will all make the same investment decision. let $x \in (0, e)$ be the amount of investment in technology 1 and let $(e_1, e_2) = (x, R(e-x))$ be the vector of $t = 1, 2$ endowments conditional on x. Let $(p_1, p_2) = (1, p)$ be the price of consumption at $t = 1, 2$. Compute type i's demand for the $t = 1, 2$ goods using p, e_1, e_2.

(c) (5 points) For notational simplicity, let $\bar{\beta} = \sum_{i=1}^{I} \pi_i \beta_i$ be the average discount factor. Conditional on x, compute the equilibrium price p.

(d) (10 points) Let $V(x, p) = \max U((x_{i1}, x_{i2})_i)$ be the agents' maximized utility conditional on short-term investment x and price p. Noting that agents choose x optimally given p, compute the equilibrium short-term investment x^*.

(e) (10 points) Suppose that the government can force the agents to choose a particular x, without interfering in the subsequent consumption markets at $t = 1, 2$. Let $p(x)$ be the price of $t = 2$ consumption conditional on x derived above. Prove that

$$\frac{d}{dx} V(x, p(x)) \bigg|_{x=x^*} > 0,$$

so welfare locally increases if the government forces the agents to invest more in the short-term investment technology.

You can detach this sheet and use as a scratch paper.