Econ 200B General Equilibrium
Midterm Exam

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Name: ______________________________

Instructions:

- Read these instructions and the questions carefully. Questions are not necessarily ordered in the order of difficulty.
- Don't start the exam until instructed.
- Turn off any electronic devices and put them in your bag.
- Don't put anything on your desk except the exam sheet, pens, pencils, eraser, and your ID card (*no* calculator). Failure to do so may be regarded as academic dishonesty.
- All logarithms are natural logarithms, i.e., base $\mathrm{e} = 2.718281828\dots$.
- "Show" or "prove" require a formal mathematical proof or argument. "Explain" only requires an intuitive, though logically coherent, verbal argument.
- This exam has 4 questions on 9 pages excluding the cover page, for a total of 100 points.
- Write the answer in the space below each question, unless otherwise stated in the question. If you don't have enough space you can use other parts of the exam sheet, but make sure to indicate where.
- You can detach the last empty page and use it as a scratch sheet.

Question:	1	2	3	4	Total
Points:	10	30	30	30	100
Score:					

1. The hyperbolic absolute risk aversion (HARA) utility function satisfies

$$-\frac{u''(x)}{u'(x)} = \frac{1}{ax+b},$$

where a, b are constants and we only consider the range of x such that $ax + b > 0$.

(a) (3 points) Derive the functional form of u (up to a monotonic transformation) when $a = 0$.

(b) (3 points) Derive the functional form of u (up to a monotonic transformation) when $a = 1$.

(c) (4 points) Derive the functional form of u (up to a monotonic transformation) when $a \neq 0, 1$.

2. Consider an economy with two goods and two agents $i = A, B$. Both agents have utility function

$$u(x_1, x_2) = \min\{x_1, x_2\}.$$

The endowments are $e_A = (1, a)$ and $e_B = (1, b)$, where $a, b > 0$. Below, normalize prices such that $p_1 + p_2 = 1$.

(a) (5 points) What is the name of this utility function?

(b) (5 points) Does this economy necessarily have an equilibrium? Explain.

(c) (5 points) If an equilibrium exists, is it necessarily Pareto efficient? Explain.

(d) (10 points) Compute all possible equilibrium prices (if they exist) and construct an equilibrium allocation for each price.

(e) (5 points) Suppose $a + b = 2$, $a > 1$, and $p_1 \in (0, 1)$. Show that if agent A destroys a small enough amount of endowment of good 2, he becomes strictly better off in equilibrium. Is this strange?

3. Consider the following general equilibrium model. There is a continuum of agents, where the population is normalized to 1. There are two types of agents indexed by $i = 1, 2$. There are three time periods indexed by $t = 0, 1, 2$. At $t = 0$, agents are born and their types are realized. Agents consume only at $t = 1, 2$, and they have identical endowments $(e_1, e_2) = (a, b)$. Type i agents have utility function

$$U_i(x_1, x_2) = (1 - \beta_i)u(x_1) + \beta_i u(x_2),$$

where $\beta_i \in (0, 1)$ is the discount factor, x_t is consumption at t, and u is a von Neumann-Morgenstern utility function. There are no markets at $t = 0$, and hence agents cannot trade assets contingent on their type realization. At $t = 1$, the economy becomes a standard Arrow-Debreu economy. Let $\pi_i > 0$ be the probability of being type i, which is also the fraction of type i agents.

(a) (5 points) Define and compute the competitive equilibrium when $u(x) = \log x$.

(b) (10 points) Consider the social planner's problem, which is to maximize the ex ante welfare

$$\mathrm{E}[U_i(x_{i1}, x_{i2})] = \sum_{i=1}^{2} \pi_i U_i(x_{i1}, x_{i2})$$

subject to the feasibility constraint. Solve the planner's problem when $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, where $\gamma > 0$. (As usual, $u(x) = \log x$ when $\gamma = 1$.)

(c) (5 points) Show that the allocations in competitive equilibrium and social planner's problem coincide if $u(x) = \log x$.

(d) (10 points) Suppose that $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ with $\gamma \neq 1$ and $\beta_1 \neq \beta_2$. Show that for generic choices of (a, b), the allocations in competitive equilibrium and social planner's problem do not coincide. More specifically, show that for any $a > 0$, there is exactly one value of $b > 0$ such that the two allocations coincide.

4. Consider the binomial option pricing model discussed in the lectures. Time is denoted by $t = 0, 1, \ldots, T$. The gross risk-free rate is constant at $R > 0$. Each period, the

stock can go up or down, so

$$S_{t+1} = \begin{cases} US_t & \text{if stock goes up,} \\ DS_t & \text{if stock goes down,} \end{cases}$$

where $U > R > D$. Suppose the initial stock price $S_0 > 0$ is given and consider an American put option with strike price K and expiration T. Recall that a put option is the right to sell a stock at the strike price, and "American" means that the option can be exercised any time until expiration.

(a) (4 points) Suppose for simplicity that $T = 1$. Let u, d stand for the up and down states and p_u, p_d be the state prices. Derive two equations that p_u, p_d satisfy.

(b) (3 points) Compute p_u, p_d.

(c) (3 points) Compute the price of the American put option.

(d) (10 points) Now suppose that the expiration date T is arbitrary. Prove that the put option price is decreasing in the current stock price S_0.

(e) (10 points) Prove that the put option price is convex in the current stock price S_0.

You can detach this sheet and use as a scratch paper.