ECONOMICS 205:
Mathematics for Economists

Summer 2023

Basic information

Lectures	M-F September 5-22, 8:30-10:30, SSB 107
Instructor	Prof. Alexis Akira Toda
Email	atoda@ucsd.edu
Webpage	`https://alexisakira.github.io/`
	(Go to `Teaching` → `Mathematics for Economists`)
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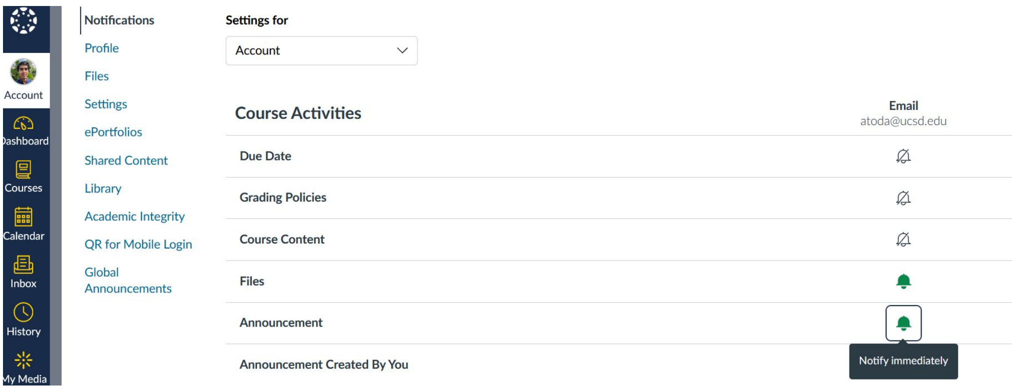
Canvas

I will be using Canvas (`https://canvas.ucsd.edu`) to communicate with you. Please access it and familiarize yourself. We do not monitor Canvas Inbox; please do not use it. Most importantly, please turn on the notification setting to receive announcement notifications. Go to `Account` → `Notifications` and select "Notify immediately" for "Announcement". See the screen shot below. Make sure to also check your spam folders. In the past, I received complaints such as "we didn't receive announcements" but this is an issue at the student's end.



Course description

Modern economics requires substantial knowledge of mathematical techniques. Most of the incoming Ph.D. students should already be familiar with linear algebra and real analysis. The course Econ 205 Mathematics for Economists (commonly known as "math camp") has two purposes. First, it quickly brushes up (for those who are familiar) or introduces (for those who are not familiar) basic mathematical concepts. Second, and more importantly, it studies in detail some mathematical topics that are important for economic analysis yet generally not covered deeply in mathematics courses or textbooks at the undergraduate level. These topics include: nonnegative matrices, convex analysis, constrained optimization, and dynamic programming.

Expectations

We will cover materials that are normally taught over a few years in just 30 hours. You need to work hard to keep up with the course and digest materials. If your mathematical preparation is limited, you need to study basic linear algebra at the level of Jänich (1994) and real analysis at the level of Rudin (1976) as soon as possible. I am happy to explain these materials if requested, but we simply have no lecture time to do it systematically.

Every day, after the lectures attempt to solve the end-of-chapter exercises on your own. If you just sit in the class, probably you will be lost. It is crucial that you solve problems in order to digest the materials.

Textbook

Lectures will be based on the draft of my textbook posted at my website. In addition, the following textbooks may be useful, though not required.

Berge (1963) Concise textbook on topological spaces, convex analysis, and some fixed point theory. Must-have for theorists.

Luenberger (1969) Beautiful textbook on introduction to functional analysis and optimization in infinite-dimensional spaces.

Kolmogorov and Fomin (1970) Introduction to topological spaces and functional analysis.

Rudin (1976) Standard textbook on real analysis.

Jänich (1994) Introductory textbook on linear algebra.

Simon and Blume (1994) Standard textbook on mathematics for economists with plenty of exercises but skips many proofs.

Sundaram (1996) Introduction to optimization but skips many proofs.

Folland (1999) Beautiful textbook on measure theory with introduction to topological spaces and functional analysis with some applications.

Lax (2007) Concise textbook on linear algebra with lots of applications and sufficient amount of pages devoted to the analytical aspect of matrices.

Horn and Johnson (2013) Standard reference book for matrix analysis (analytical aspect of linear algebra). Must-have for theorists and econometricians. The 1985 first edition is more readable (the second edition is too encyclopedic).

Course outline

The course closely follows the draft of my textbook. I will prepare slides that cover the essential materials and explain the key ideas. However, this is a proof-based mathematics course, not an economics course, and to master the materials, it is essential that you actually go through the proofs. Thus, after attending lectures and looking at the slides, you should read proofs and attempt to solve end-of-chapter exercises.

Below is a short list of topics that I intend to cover (roughly in this order).

- Introduction to optimization
  - existence of solutions (basic topology in $\mathbb{R}^N$, continuous functions, extreme value theorem): Ch. 1
  - unconstrained optimization with one or multiple variables (differentiation, chain rule, first-order condition): Ch. 2, 3
  - introduction to constrained optimization (intuitive understanding of the Karush-Kuhn-Tucker theorem): Ch. 4
- Linear algebra
  - basic linear algebra (matrices as representation of linear maps, solvability of linear equations, determinant): Ch. 5
  - spectral theory (eigenvalues and eigenvectors, diagonalization and upper triangularization of matrices, symmetric matrices, matrix norm, spectral radius): Ch. 6
- Convex analysis
  - convex sets (definition, separating hyperplane theorem): Ch. 9
  - convex functions (convex and quasi-convex functions, convexity-preserving operations, differential characterization): Ch. 10
  - convex programming (saddle point theorem, KKT theorem for convex and quasi-convex programming, duality): Ch. 11
- Nonlinear programming (general KKT theorem, constraint qualification, parametric continuity and differentiability): Ch. 12
- Dynamic programming
  - examples in finite-horizon: Ch. 13
  - infinite-horizon dynamic programming (metric space, contraction mapping theorem, value function iteration): Ch. 7, 14
- Other topics
  - implicit function theorem: Ch. 7
  - Markov chain and nonnegative matrices: Ch. 8

If you master these topics, I think you will be in a good shape to start doing research in economics.

If you plan to be a theorist or an econometrician, I also recommend you to study measure theory, probability theory, and functional analysis. Maybe there are Ph.D. level courses at the mathematics department but I am not particularly familiar.

Exam

The final is on Monday September 25. The format is to be determined.

Questions

The best opportunity to ask questions is *during* the class, for two reasons. First, you can resolve your question immediately (assuming—well—I know the answer). Second, your classmates are likely to have similar questions, so they can benefit from questions being resolved and I benefit by saving time. So, don't be shy, please ask questions.

References

Claude Berge. *Topological Spaces.* Oliver & Boyd, Edinburgh, 1963.

Gerald B. Folland. *Real Analysis: Modern Techniques and Their Applications.* John Wiley & Sons, Hoboken, NJ, 2 edition, 1999.

Roger A. Horn and Charles R. Johnson. *Matrix Analysis.* Cambridge University Press, New York, 2 edition, 2013.

Klaus Jänich. *Linear Algebra.* Undergraduate Texts in Mathematics. Springer, New York, 1994. doi: 10.1007/978-1-4612-4298-7.

A. N. Kolmogorov and S. V. Fomin. *Introductory Real Analysis.* Prentice-Hall, 1970. Translated and edited by R. A. Silverman.

Peter D. Lax. *Linear Algebra and Its Applications.* John Wiley & Sons, Hoboken, NJ, 2 edition, 2007.

David G. Luenberger. *Optimization by Vector Space Methods.* John Wiley & Sons, New York, 1969.

Walter Rudin. *Principles of Mathematical Analysis.* McGraw-Hill, 3 edition, 1976.

Carl P. Simon and Lawrence E. Blume. *Mathematics for Economists.* W. W. Norton & Company, New York, 1994.

Rangarajan K. Sundaram. *A First Course in Optimization Theory.* Cambridge University Press, NY, 1996.