Consumption-based Asset Pricing

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1 Lucas (1978) model

Unlike financial practitioners, whose goal is to profit from arbitrage, economists are interested in why asset prices are what they are. Lucas (1978) has provided a framework to compute asset prices given the fundamentals of the economy, which is now called *consumption-based (capital) asset pricing model*, or cCAPM.

Consider an economy with multiple periods denoted by $t = 0, 1, \dots$. Consider an asset that pays dividend D_t at time t, which is a random variable (adapted stochastic process). Let P_t be the ex-dividend price of this asset. Letting m_t be the stochastic discount factor at time t, by no-arbitrage we know that it must be

$$P_t = \mathrm{E}_t[m_{t+1}(P_{t+1} + D_{t+1})].$$

Let us define the marginal utility process $\{\Lambda_t\}_{t=0}^\infty$ as follows. Take any Λ_0 and define $\Lambda_t = \Lambda_0 \prod_{s=1}^t m_s = \Lambda_{t-1} m_t$. From the above equation, we obtain

$$\Lambda_t P_t = \mathrm{E}_t[\Lambda_{t+1}(P_{t+1} + D_{t+1})].$$

Iterating this equation, using the law of iterated expectations, and assuming the no bubble condition $\lim_{T\to\infty} \mathrm{E}_t[\Lambda_T P_T] = 0$, it follows that

$$\Lambda_t P_t = \sum_{n=1}^\infty \mathrm{E}_t[\Lambda_{t+n} D_{t+n}]$$

$$\iff P_t = \frac{1}{\Lambda_t} \sum_{n=1}^\infty \mathrm{E}_t[\Lambda_{t+n} D_{t+n}].$$

Thus, if we know the data generating process of $\{\Lambda_t, D_t\}_{t=0}^\infty$, in principle we can compute the asset price.

Now suppose that an investor has an additive utility function

$$\mathrm{E}_0 \sum_{t=0}^\infty \beta^t u(c_t),$$

where $\beta > 0$ is the discount factor and c_t is consumption. What is the stochastic discount factor in this case? For simplicity, suppose that there are only two time

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periods, $t = 0, 1$. Letting π_s be the (objective) probability of state s and p_s be the price of the state-s Arrow security, the investor's problem is

$$\begin{aligned} &\text{maximize} && u(c_0) + \sum_{s=1}^{S} \pi_s \beta u(c_s) \\ &\text{subject to} && c_0 + \sum_{s=1}^{S} p_s c_s \le w, \end{aligned}$$

where w is initial wealth and the price of consumption at $t = 0$ is normalized to 1. The Lagrangian of this optimization problem is

$$L = u(c_0) + \sum_{s=1}^{S} \pi_s \beta u(c_s) + \lambda \left(w - c_0 - \sum_{s=1}^{S} p_s c_s \right).$$

The first-order condition with respect to c_0 and c_s are

$$\begin{aligned} 0 &= u'(c_0) - \lambda, \\ 0 &= \pi_s \beta u'(c_s) - \lambda p_s, \end{aligned}$$

respectively. Eliminating λ and using the definition of the stochastic discount factor, we obtain

$$m_s = \frac{p_s}{\pi_s} = \beta \frac{u'(c_s)}{u'(c_0)}.$$

Therefore if we choose $\Lambda_0 = u'(c_0)$, then $\Lambda_t = \beta^n u'(c_t)$, so the asset price becomes

$$P_t = \mathrm{E}_t \sum_{n=1}^{\infty} \beta^n \frac{u'(c_{t+n})}{u'(c_t)} D_{t+n}.$$

If $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ (CRRA), letting $X_t = (\log c_t, \log D_t)'$, and $\alpha = (-\gamma, 1)'$, we obtain

$$\begin{aligned} \frac{P_t}{D_t} &= \mathrm{E}_t \sum_{n=1}^{\infty} \beta^n \left(\frac{c_{t+n}}{c_t} \right)^{-\gamma} \frac{D_{t+n}}{D_t} \\ &= \mathrm{E}_t \sum_{n=1}^{\infty} \beta^n \mathrm{e}^{\alpha'(X_{t+n} - X_t)} \\ &= \sum_{n=1}^{\infty} \beta^n M_{X_{t+n} - X_t}(\alpha), \end{aligned} \tag{1}$$

where $M_{X_{t+n}-X_t}$ is the moment generating function (MGF) of $X_{t+n} - X_t$. Therefore, given the discount factor β, risk aversion γ, and the stochastic process of log consumption and dividend, in principle we can compute the price-dividend ratio of the asset.

Of course, in order for this approach to be operational, one has to know the individual consumption. Some researchers have adopted (heroic) assumptions to get around this issue. First, assume that all agents have identical CRRA utility functions and markets are complete. In this case we can treat the economy as if a representative agent consumes the aggregate endowment, so we can use aggregate consumption C_t instead of individual consumption c_t. Second, for simplicity assume that dividend equals aggregate consumption, so $C_t = D_t$. Letting $M_{t,n}$ be the moment generating function of log consumption growth $\log(C_{t+n}/C_t)$, it follows that

$$\frac{P_t}{C_t} = \sum_{n=1}^{\infty} \beta^n M_{t,n}(1-\gamma). \tag{2}$$

As an example, suppose that log consumption growth $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is i.i.d. over time. Letting $M(s) = \mathrm{E}[\mathrm{e}^{s\Delta c}]$ be the moment generating function, we obtain

$$\frac{P_t}{C_t} = \sum_{n=1}^{\infty} \beta^n (M(1-\gamma))^n = \frac{\beta M(1-\gamma)}{1-\beta M(1-\gamma)},$$

assuming $\beta M(1-\gamma) < 1$. (Otherwise the price-dividend ratio is infinite.)

2 Closed-form solutions to asset pricing models

Recall the general formulas for the price-dividend or price-consumption ratios (1), (2). In these formulas, the price-dividend ratio is the discounted sums of moment generating functions of log consumption/dividend growth. For specific consumption/dividend growth processes, we may be able to obtain closed-form solutions to asset pricing models. For example, since the moment generating function of $N(\mu, \sigma^2)$ is $M(s) = \mathrm{e}^{\mu s + \frac{1}{2}\sigma^2 s^2}$, we can solve for the price-dividend ratio in closed form if consumption growth is lognormal. However, there are many more examples.

Burnside (1998) obtained a solution when log consumption/dividend growth follows a Gaussian VAR(1) process. This model is useful for evaluating the accuracy of discretization of VARs (see Farmer and Toda (2017)). Tsionas (2003) generalized to the case with non-Gaussian shocks, and de Groot (2015) to the case with stochastic volatility.

Yet another example is when the log consumption/dividend growth follows a finite-state Markov chain. Suppose that there are S states indexed by $s = 1, \dots, S$, and let $P = (p_{ss'})$ be the transition probability matrix. Let $X_t = (\log(C_t/C_{t-1}), \log(D_t/D_{t-1}))'$ be the vector of log consumption and dividend growth.

Using this notation, we can obtain closed-form solutions as follows. By the Euler equation, we have

$$P_t = \mathrm{E}_t[\beta(C_{t+1}/C_t)^{-\gamma}(P_{t+1} + D_{t+1})]. \tag{3}$$

Dividing (3) by D_t, we obtain

$$V_t = \beta \, \mathrm{E}_t[\exp(\alpha' X_{t+1})(V_{t+1} + 1)], \tag{4}$$

where $\alpha = (-\gamma, 1)'$. Since the process for consumption and dividend growth is a Markov chain, letting x_s be the vector of log consumption/dividend growth in state s, (4) becomes

$$v_s = \beta \sum_{s'=1}^{S} p_{ss'} \mathrm{e}^{\alpha' x_{s'}} (v_{s'} + 1), \tag{5}$$

where v_s is the price-dividend ratio in state s. Let $v = (v_1, \dots, v_S)'$ $(S \times 1)$ and $X = (x_1, \dots, x_S)'$ $(S \times 2)$ be the matrices of those values. Then (5) is equivalent to the linear equation

$$v = \beta P \operatorname{diag}(\mathrm{e}^{X\alpha})(v + 1) \iff v = (I - \beta P \operatorname{diag}(\mathrm{e}^{X\alpha}))^{-1} \beta P \mathrm{e}^{X\alpha},$$

so we obtain a closed-form solution. Such a solution has been used by many authors, for example Mehra and Prescott (1985), Cecchetti et al. (1993), Bonomo et al. (2011), among others.

3 Asset pricing puzzles

Let us compute the risk-free rate, expected stock returns, and equity premium under the assumptions of CRRA representative agent and i.i.d. consumption growth. Since the stochastic discount factor is $\beta(C_{t+1}/C_t)^{-\gamma}$, the gross risk-free rate R_f satisfies

$$\frac{1}{R_f} = \mathrm{E}[\beta(C_{t+1}/C_t)^{-\gamma}] = \beta M(-\gamma) \iff R_f = \frac{1}{\beta M(-\gamma)}.$$

Since the gross stock return is

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} = \frac{C_{t+1}}{C_t} \frac{P_{t+1}/C_{t+1} + 1}{P_t/C_t} = \frac{C_{t+1}}{C_t} \frac{1}{\beta M(1-\gamma)},$$

taking expectations, the expected stock return is

$$\mathrm{E}[R] = \frac{M(1)}{\beta M(1-\gamma)}.$$

Therefore the log equity premium is

$$\log \mathrm{E}[R] - \log R_f = \log \frac{M(1) M(-\gamma)}{M(1-\gamma)}.$$

Intuitively, the equity premium should be nonnegative. Indeed, we can prove that that is the case.

Proposition 1. *Let M be a moment generating function. Then*

$$\log M(1) + \log M(-\gamma) \geq \log M(1-\gamma).$$

Proof. Let X be log consumption growth and take any s_1, s_2 and $t \in (0,1)$. Letting $p = \frac{1}{1-t}$ and $q = \frac{1}{t}$, we have $1/p + 1/q = 1$. By Hölder's inequality, we obtain

$$\begin{aligned} M((1-t)s_1 + ts_2) &= \mathrm{E}[\mathrm{e}^{((1-t)s_1 + ts_2)X}] = \mathrm{E}[\mathrm{e}^{(1-t)s_1 X} \mathrm{e}^{ts_2 X}] \\ &\leq \mathrm{E}[(\mathrm{e}^{(1-t)s_1 X})^p]^{\frac{1}{p}} \, \mathrm{E}[(\mathrm{e}^{ts_2 X})^q]^{\frac{1}{q}} \\ &= \mathrm{E}[\mathrm{e}^{s_1 X}]^{1-t} \, \mathrm{E}[\mathrm{e}^{s_2 X}]^t \\ &= M(s_1)^{1-t} M(s_2)^t. \end{aligned}$$

Taking the logarithm, we obtain

$$\log M((1-t)s_1 + ts_2) \le (1-t)\log M(s_1) + t\log M(s_2),$$

so $\log M$ is convex (M is log-convex). For notational simplicity, let $f(s) = \log M(s)$. Since $\gamma > 0$ we have $-\gamma < 0, 1-\gamma < 1$. Since

$$0 = (1-t)(-\gamma) + t \iff t = \frac{\gamma}{\gamma+1},$$
$$1-\gamma = (1-t)(-\gamma) + t \iff t = \frac{1}{\gamma+1},$$

By the convexity of f we obtain

$$f(0) \le \frac{1}{\gamma+1} f(-\gamma) + \frac{\gamma}{\gamma+1} f(1),$$
$$f(1-\gamma) \le \frac{\gamma}{\gamma+1} f(-\gamma) + \frac{1}{\gamma+1} f(1).$$

Adding these two inequalities and noting that $f(0) = \log M(0) = 0$ because $M(0) = 1$, it follows that $f(1-\gamma) \le f(-\gamma) + f(1)$, which is the conclusion. □

As a concrete example, assume that consumption growth is lognormal, so $\Delta c_{t+1} \sim N(\mu, \sigma^2)$. The moment generating function is

$$M(s) = \mathrm{e}^{\mu s + \frac{1}{2}\sigma^2 s^2}.$$

Therefore the log risk-free rate is

$$\log R_f = -\log\beta + \mu\gamma - \frac{1}{2}\sigma^2\gamma^2, \tag{6}$$

and the log equity premium is

$$\begin{aligned}&\log \mathrm{E}[R] - \log R_f \\ &= \left(\mu + \frac{1}{2}\sigma^2\right) + \left(-\mu\gamma + \frac{1}{2}\sigma^2\gamma^2\right) - \left(\mu(1-\gamma) + \frac{1}{2}\sigma^2(1-\gamma)^2\right) = \gamma\sigma^2.\end{aligned}$$

As a numerical illustration, using the data from Mehra and Prescott (1985), let us assume that log consumption growth volatility is 3.57% per year, and the equity premium is 6.18% per year. Then it must be the case that

$$0.0618 = \gamma \times 0.0357^2 \iff \gamma = 48.49,$$

which is quite high. This is the equity premium puzzle: Mehra and Prescott (1985) argue that a reasonable upper bound for the relative risk aversion is 10, and with this bound the model cannot generate a realistic equity premium.

Why is an upper bound of $\gamma \le 10$ reasonable? What is wrong with turning up γ beyond 10? According to the risk-free rate formula (6), we have

$$\log R_f = -\log\beta + \frac{\mu^2}{2\sigma^2} - \frac{1}{2}\sigma^2\left(\gamma - \frac{\mu}{\sigma^2}\right)^2,$$

which is a quadratic function in γ and attains maximum at $\gamma = \mu/\sigma^2$. Thus if we increase γ in order to make the equity premium larger, we will also make the risk-free rate larger (up to some point). However, historically the risk-free rate has been low, say 1%. Therefore turning up risk aversion cannot explain the low risk-free rate, which is known as the risk-free rate puzzle (Weil, 1989). Then how about turning up the risk aversion even further? Since the log risk-free rate (6) is quadratic, $\log R_f$ will eventually be decreasing in γ. The issue now is that the log risk-free rate will be extremely sensitive to economic growth. By (6), we have

$$\frac{\partial \log R_f}{\partial \mu} = \gamma.$$

If, for example, $\gamma = 100$, then a 1% change in economic growth (μ) will translate into 100% change in the risk-free rate. However, historically the risk-free rate is quite stable. See Kocherlakota (1996) for an early review of the equity premium puzzle.

Of course, in order for the equity premium puzzle to be a puzzle, one has to maintain many assumptions, such as CRRA utility, representative agent, consumption equals dividend, complete markets, i.i.d. consumption growth, lognormal consumption growth, etc. The literature has naturally relaxed some of these assumptions to explain asset pricing puzzles.[1] Below are such examples.

4 Some explanations of asset pricing puzzles

4.1 Rare disasters

Perhaps the puzzle arises from the Gaussian assumption. One of the earliest explanations of the equity premium puzzle is the rare disasters model (Rietz, 1988). Initially this model was dismissed as unrealistic, but it was revived by Barro (2006), who collected international data to calibrate parameters, and has become quite popular recently.

For simplicity, suppose that aggregate consumption grows at a constant rate μ when there is no disaster, and decreases a lot when hit by a disaster. Formally, we have

$$\frac{C_{t+1}}{C_t} = \mathrm{e}^{\mu} \times \begin{cases} 1, & \text{(no disaster, with probability } 1-p) \\ b, & \text{(disaster, with probability } p) \end{cases}$$

where $0 < b < 1$ is the size of downward jump after disaster. The moment generating function of log consumption growth is

$$M(s) = \mathrm{E}[\mathrm{e}^{s\Delta c}] = \mathrm{E}[(C_{t+1}/C_t)^s] = \mathrm{e}^{\mu s}(1 - p + pb^s).$$

Therefore the log equity premium is

$$\begin{aligned} \log \frac{\mathrm{E}[R]}{R_f} &= \log(1 - p + pb) + \log(1 - p + pb^{-\gamma}) - \log(1 - p + pb^{1-\gamma}) \\ &\approx p(-1 + b + (-1 + b^{-\gamma}) - (-1 + b^{1-\gamma})) \\ &= p(1 - b)(b^{-\gamma} - 1), \end{aligned}$$

[1] Economists will never agree on the explanation of asset pricing puzzles (at least in the near future), because if they agree on the explanation, they can no longer write papers on the topic. Thus puzzles must remain as puzzles until they retire.

where I have used the approximation $\log(1+pa) \approx pa$ when $p \ll 1$. (This approximation is exact in continuous-time models.) To make a fair comparison with the Gaussian model, let us compute the variance of log consumption growth. Under rare disasters, since $\Delta c = \mu$ with probability $1-p$ and $\Delta c = \mu - \log b$ with probability p, the variance is $\sigma^2 = p(1-p)(\log b)^2$. Noting that $0 < b < 1$, the volatility is then $\sigma = -\sqrt{p(1-p)} \log b$.

Collecting international data, Barro (2006) argues that $p = 0.017$. Assuming $\sigma = 0.0357$, it must be $b = 0.759$, or $100(1-b) = 24\%$ output loss after the disaster. With this number, setting $\gamma = 10$, the log equity premium becomes 0.0608, or 6.08%, which is very close to the data. So, moving away from the Gaussian assumption goes a long way towards explaining the equity premium. Rare disasters model have become quite popular these days: see, for example, Gourio (2012), Gabaix (2012), and Wachter (2013). See Barro and Ursúa (2012) for some introduction.

4.2 Incomplete markets

Perhaps the puzzle arises from the complete market assumption. In reality, markets are incomplete: existing assets do not span all states of the world. To convince you that markets are incomplete, suppose that markets were indeed complete. Then you would be able to sell off all your future labor income. If you were able to do that, you might lose the incentive to work.

Constantinides and Duffie (1996) is a typical example of an asset pricing model with incomplete markets. They show that, given any set of assets with no arbitrage and a stochastic process for the aggregate consumption growth, you can construct individual endowment processes that are consistent with the asset prices. The model is as follows.

Let C_t be aggregate consumption growth, and Λ_t be the marginal utility process that is consistent with the asset prices. (By no-arbitrage asset pricing, such Λ_t always exists.) Suppose that there are a continuum of agents indexed by $i \in [0,1]$, all having additive utility function with discount factor β and relative risk aversion $\gamma > 0$. Let c_{it} be agent i's consumption at time t. Suppose that $c_{i0} = C_0$, and define c_{it} $(t \geq 1)$ by

$$\frac{c_{it}}{C_t} = \frac{c_{i,t-1}}{C_{t-1}} \mathrm{e}^{\sigma_t \eta_{it} - \frac{1}{2}\sigma_t^2},$$

where $\sigma_t > 0$ is the standard deviation of individual consumption growth (relative to the aggregate consumption growth), to be specified later, and η_{it} is a standard normal variable that is i.i.d. across agents and over time. Note that $\mathrm{E}_t[\mathrm{e}^{\sigma_t \eta_{it} - \frac{1}{2}\sigma_t^2}] = 1$ using the property of the normal distribution, so we obtain

$$\frac{\mathrm{E}_t[c_{it}]}{C_t} = \frac{\mathrm{E}_{t-1}[c_{i,t-1}]}{C_{t-1}}.$$

Since $c_{i0} = C_0$, by induction we have $\mathrm{E}_t[c_{it}] = C_t$, so the cross-sectional average consumption is indeed the aggregate consumption.

Now we specify σ_t to support no-arbitrage pricing. The individual stochastic discount factor from time $t-1$ to t is

$$M_{it} = \beta \left(\frac{c_{it}}{c_{i,t-1}} \right)^{-\gamma} = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\gamma} \mathrm{e}^{-\gamma \sigma_t \eta_{it} + \frac{1}{2}\gamma \sigma_t^2}.$$

Since consumption growth has an idiosyncratic component which is independent of all asset payoffs, we can take expectations with respect to the idiosyncratic shock to obtain a new SDF:

$$\frac{\Lambda_t}{\Lambda_{t-1}} = \mathrm{E}_t[M_{it}] = \beta \left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \mathrm{e}^{\frac{1}{2}\gamma(\gamma+1)\sigma_t^2}.$$

Solving for σ_t, we can make this equation true if

$$\sigma_t = \sqrt{\frac{2}{\gamma(\gamma+1)}} \left(-\log\beta + \gamma \log\frac{C_t}{C_{t-1}} + \log\frac{\Lambda_t}{\Lambda_{t-1}}\right)^{\frac{1}{2}}.$$

The result of Constantinides and Duffie (1996) is a possibility theorem, and whether the idiosyncratic shock in individual consumption growth can explain asset prices or not is an empirical/quantitative question. The main papers in this literature are Brav et al. (2002), Cogley (2002), Balduzzi and Yao (2007), Storesletten et al. (2007), and Krueger and Lustig (2010). See Toda and Walsh (2015, 2017) for related issues.

4.3 Habit formation

Perhaps the puzzle arises from the CRRA assumption. Campbell and Cochrane (1999) consider a representative agent model with utility function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma},$$

where C_t is aggregate consumption and X_t is the "habit" level (reference point). Let $S_t = \frac{C_t - X_t}{C_t}$ be the "surplus consumption ratio". The stochastic discount factor is then

$$M_{t+1} = \beta \left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t}\right)^{-\gamma} = \beta \left(\frac{S_{t+1}}{S_t}\right)^{-\gamma} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

As in the case with Constantinides and Duffie (1996), it is obvious that by playing around with S_t, the model can replicate any stochastic discount factor (Pohl (2016) formally proves this). A weakness of this model is that S_t is not observable and therefore the model is not falsifiable.

4.4 Long-run risk

Perhaps the puzzle arises from the i.i.d. assumption. The long-run risk model of Bansal and Yaron (2004) and Bansal et al. (2012) combine small but persistent risk (long-run risk) and the preference for the early resolution of uncertainty to generate large equity premia. We will consider such models later.

4.5 Heterogeneous preferences

Perhaps the puzzle arises from the representative agent assumption (or identical preferences). Gârleanu and Panageas (2015) consider a model with i.i.d. consumption growth, but with two types of agents (one risk tolerant, the other risk averse). We will consider such models later.

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