Report on "Rational Bubbles: A Clarification"

Jianjun Miao*and Pengfei Wang

September 14, 2024

1 Summary of the Submission

This is a comment paper on Miao and Wang (2018, AER), henceforth MW for short. In this paper, the authors prove the "nonexistence" of "rational bubbles" in the MW model based on their definition of rational bubbles. They also aim to clarify the precise mathematical definition and the economic meaning of "rational bubble" in an accessible way to the general audience.

In this report, we refer to the comment paper by Hirano and Toda (2024) as HT for short.

2 Comments

HT do not find any mathematical errors or logical problems with the economic mechanisms in Miao and Wang (2018). Their comment focuses exclusively on the use of the terminology "rational bubbles." In our view, their comment paper does not provide any new insights. First, their definition of rational bubbles is taken from the textbook by Miao (2014), who is one of the authors of Miao and Wang (2018). This definition is based on the seminal work of Santos and Woodford (1997). Second, the Bubble Characterization Lemma is taken from Montrucchio (2004) and is not the original contribution by HT. This lemma essentially provides a simple condition to check the transversality condition. HT use this lemma to explain the "fundamental difficulty in attaching rational bubbles to dividend-paying assets." This difficulty has already been explained by Miao and Wang (2018, Section I) in a different and more intuitive way. Finally, in their Proposition 1, HT use the Bubble Characterization Lemma to show the "nonexistence" of rational bubbles in Miao and Wang (2018). The proof is trivial.

*Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.

In fact, HT do not need to use the Bubble Characterization Lemma or Proposition 1 to show the "nonexistence" of "rational bubbles" in MW (2018) because MW already point out that the usual transversality condition holds in their model. Let us use the simple discrete time setting in Miao and Wang (2018, Section 1) to explain the intuition. The transversality condition is

$$\lim_{T\to\infty} e^{-rT} V_{t+T} = 0, \tag{1}$$

where $r > 0$ is the discount rate and V_t denotes the cum-dividends stock price, which satisfies the pricing equation

$$V_t = D_t + e^{-r} V_{t+1}, \tag{2}$$

where $D_t > 0$ denotes dividends. Equation (1) corresponds to (5) in the HT paper. By the traditional definition (which is also stated in their comment paper), there is no "rational bubble" at time t if and only if (1) holds.

To understand the last claim, we can decompose V_t into

$$V_t = V_t^* + B_t,$$

where V_t^* is the "fundamental component" satisfying

$$V_t^* = \sum_{s=0}^{\infty} e^{-rs} D_{t+s}, \tag{3}$$

and $B_t \equiv \lim_{T\to\infty} e^{-rT} V_{t+T}$ is the bubble component satisfying

$$B_t = e^{-r} B_{t+1}. \tag{4}$$

Thus the transversality condition (1) implies $B_t = 0$ and rules out bubbles.

Notice that the traditional theory summarized above (or the exposition given in the HT paper) is derived from the asset pricing equation (2) alone without any additional constraints. In particular, the pricing equation (4) for the bubble B_t naturally follows from (2).

By contrast, in the model of Miao and Wang (2018), dividends D_t are endogenously generated in a production economy with endogenous credit constraints so that B_t follows a pricing equation different from (4). MW show that equation (2) takes a form of the Bellman equation subject to the endogenous credit constraints and several additional constraints. They show that the stock price (or the stock market value of the firm) can be decomposed into

$$V_t = Q_t K_t + B_t, \tag{5}$$

where $Q_t K_t$ is the fundamental component as is well known in the standard Tobin's Q theory and B_t satisfies

$$B_t = e^{-r} B_{t+1} \left(1 + LIQ_{t+1}\right), \tag{6}$$

where $LIQ_{t+1} > 0$ represents a liquidity premium. MW show that a pure bubble asset also follows the same pricing equation (6), which resembles the money bubble equation in the monetary theory literature surveyed by Lagos et al (2017). Thus, MW interpret B_t as a bubble component, which will be discussed further below. They also show that the transversality condition (1) cannot rule out bubbles in (6) in the sense that the model features multiple equilibria. One type of equilibria is the fundamental equilibrium with $B_t = 0$ for all t. The other type is the bubbly equilibrium with $B_t > 0$ for all t. There are also other types, e.g., the stochastic bubble equilibrium. There are two steady states with $B = 0$ and $B > 0$. The bubbleless steady state with $B = 0$ has indeterminacy of degree 1 and the bubbly steady state with $B > 0$ is a saddle point. All these features resemble those in the bubble literature, e.g., Tirole (1985) and Weil (1987).

We think the main differences of opinions between us and HT lie in two key issues: (i) the definition of rational bubbles and (ii) the interpretation of B_t in (6). We now discuss these two issues and other related issues.

Definition of Rational Bubbles. It is intuitive and generally agreed that an asset bubble is defined as the difference between the asset market price and its fundamental value. The main difficulty is how to define fundamental value. As Tirole (1985, p. 1091) points out, "in some cases, the usual notion of market fundamental and bubble is not fully satisfactory." He then discusses two reasons why it may be so. The first is related to an illusion in bubble accounting. Tirole (1985) gives an example in which the fundamental value itself is a bubble. The second is related to the distinction between the financial market fundamental and the real market fundamental. Both reasons are relevant for the MW model.

First, dividends are endogenous in the MW model. In footnote 1 of MW (2018), they point out:

"A stock price bubble is defined as the difference between a stock's market value and its fundamental value, e.g., the discounted value of exogenously given dividends in exchange economies (Santos and Woodford 1997). It is subtle to apply this definition to our model because dividends are endogenously generated through investment and production and because bubbles help generate dividends."

In the MW model, a firm's borrowing capacity is limited by its own market value. If all agents speculate that firm value contains a bubble component B_t, then the firm can borrow more funds, which can finance more investment, increase dividend payment, and raise firm value. This higher firm value then supports the initial belief that firm value contains a bubble. Because of this positive feedback loop mechanism, B_t satisfies the pricing equation (6). Unlike the traditional pricing equation (4), the bubble component commands a liquidity premium LIQ_{t+1}.

Second, the component $Q_t K_t$ in firm value (5) can be intuitively interpreted as the real fundamental value because this is the traditional fundamental value measured in the Tobin's Q theory of the firm.

We should add a third reason for the difficulty of the traditional definition of the fundamental value. The traditional definition relies on the no-arbitrage pricing equation (2), or more generally, equation (1) in HT:

$$P_t = \frac{1}{\pi_t} E_t \left[\pi_{t+1} \left(P_{t+1} + D_{t+1} \right) \right], \tag{7}$$

where π_t is a state price deflator.[1] Then we have the decomposition,

$$P_t = \lim_{T\to\infty} \frac{1}{\pi_t} E_t \left[\sum_{s=t+1}^{T} \pi_s D_s \right] + \underbrace{\lim_{T\to\infty} \frac{E_t \left[\pi_T P_T \right]}{\pi_t}}_{B_t}, \tag{8}$$

$$B_t = E_t \left[\frac{\pi_{t+1}}{\pi_t} B_{t+1} \right], \tag{9}$$

The first component is the fundamental value and the second component B_t represents the traditional bubble.

The problem is that in many models with market frictions, the no-arbitrage pricing equation (7) may not hold. This equation holds in models with debt (or solvency) constraints and with

[1] There may exist multiple state price deflators under incomplete markets. See Santos and Woodford (1997) for a discussion.

borrowing constraints (e.g., Miao (2014)). But it does not hold in models with short-sales constraints. LeRoy and Werner (2001) give examples in which there exists arbitrage in models with short-sale constraints. He and Modest (1995) derive asset pricing equations different from (7) in the presence of transactions costs or other trading constraints. One can also give many other examples of models with endogenous borrowing constraints for which the asset pricing equation takes a different form than (7) (e.g., leverage/collateral constraints of Geanakopolos (2003) and Fostel and Geanakoplos (2008), and margin constraints of Garleanu and Pedersen (2011)). For these models, the fundamental value as the present value may not be well defined because the state price π_t or the stochastic discount factor is nontrivial to specify. In particular, the law of one price may fail in these models.

In summary, we think using (3) or (8) to define the fundamental value and the rational bubble has limitations. Relying on this definition alone is dogmatic and mechanical, and is harmful for the developments of new theory. We agree with HT that there are many notions of bubbles in the literature. In our view, for rational bubbles, rationality is important in the sense that all agents must have rational expectations. There is no behavioral bias or heterogeneous beliefs. A rational bubble is not backed by any fundamental payoffs and thus satisfies a pricing equation like (4) or (9). It may also satisfy equation like (6) due to the liquidity premium (e.g., the MW model and the Lagos-Wright model). These equations reveal that if $B_{t+1} = 0$, then $B_t = 0$, unlike the stock price equation (2) or (7), for which $P_{t+1} = 0$ does not imply $P_t = 0$ due to the presence of dividends $D_t > 0$ for all t. Moreover, speculation is important for the emergence of a rational bubble. If all agents believe or speculate that the asset contains a bubble ($B_{t+1} > 0$), it can be circulated and the bubble may be supported in equilibrium ($B_t > 0$) under certain conditions. The model of Miao and Wang (2018) has all these features.

Interpretation of B_t in (6). There is no confusion for the definition of a pure rational bubble like money because its fundamental value is zero. Then a positive value of money (or an intrinsically useless asset) represents a bubble. But even for the money bubble, its pricing equation may not satisfy (4), e.g., in the monetary theory of Lagos Rocheteau, and Wright (2017). In Section V. A of Miao and Wang (2018), they introduce a pure bubble asset like money. They show that the pricing equation satisfies an equation like (6) (see their Proposition 8 on page 2614). The transversality condition cannot rule out the pure bubble due to the presence of the liquidity premium.

A key contribution of Miao and Wang (2018) is to show that the component B_t in the stock price (5) follows the same pricing equation (6) for the money bubble, which is different from the traditional equation (4). Based on this fact, it is reasonable to call B_t a bubble component, just like the pure money bubble. Moreover, equation (6) has an intuitive interpretation based on speculation. There is no fundamental payoffs to support (6). If all agents believe that there is no future value for the bubble component, $B_{t+1} = 0$, then it has no value today $B_t = 0$. Thus, a bubbleless equilibrium with $B_t = 0$ for all t can exist. But if all agents speculate that there is future value $B_{t+1} > 0$, then the value can be supported as $B_t > 0$. The bubble component cannot be ruled out by the transversality condition, because it has an additional benefit by providing liquidity to the firm. The liquidity premium LIQ_{t+1} is captured in (6).

On page 10, HT claim that MW's interpretation of B_t as a bubble component is unjustified. Their accusation is baseless as MW have an intuitive discussion in Section I and the comparison with the pure bubble asset in Section V. A.

On page 12, HT claim that

"Whatever definition we adopt, we need to be consistent. It is not right to claim that a particular model generates rational bubbles attached to dividend-paying assets using one definition, when in fact there exist no rational bubbles in the same model under a different (and more standard) definition, without acknowledging the differences in the definitions."

These statements are baseless. As we discussed above, in the MW model, both the pure bubble (e.g., money) and the stock price bubble B_t satisfy the same asset pricing equation like (6). They are perfect substitutes (see Proposition 8 of MW). For consistency mentioned by HT, why do they say one is a rational bubble, but the other is not? Do they have double standards?

Bubble Characterization Lemma. HT use the Bubble Characterization Lemma to show the "nonexistence" of stock price bubble in Miao and Wang (2018). This lemma is basically an implication of the transversality condition for the stock price that satisfies pricing equation (7) in endowment economies. This lemma can rule out bubbles that satisfy equation (9) or (4). But it cannot rule out bubbles that satisfy equation (6) as in the MW model. In the MW model, bubbles allow a firm to borrow more funds so that it can pay more dividends.

Their claim on page 10 that "bubbles in dividend-paying assets can never occur so long as the price-dividend ratio (or the dividend yield) converges to a positive constant" has qualifications. The qualification is that the asset pricing equation takes the form like (7) and the bubbles they defined must satisfy a pricing equation like (9), because the Bubble Characterization Lemma is based on these two equations (see Montrucchio (2004)). If one of the equations does not hold, this lemma may not apply.

For the same reason, on page 10, HT claim that "in all papers in Table 1, no rational bubbles exist. Thus, the way Miao and Wang (2018) write their paper is misleading as they claim the existence of a bubble as if it were a rational bubble" is incorrect. Their claims reflect their misunderstanding of the MW model and their narrow understanding of rational bubbles.

Broader Economic Implications. On page 11, HT claim that

"It is more appropriate to interpret Miao and Wang (2018) and others as multiple equilibria in asset pricing models, where there are two steady states, one with high stock prices and the other with low stock prices. In both steady states, stock prices always reflect fundamentals, but self-fulfilling expectations determine which steady state is reached. In fact, Miao and Wang (2015, p. 772) state 'one may also interpret it as a self-fulfilling component or a belief component if one wants to avoid using the term 'bubble.'"

In fact, the MW model has features closer to the rational bubble model of Tirole (1985). For example, there are two steady states: one is bubblebless with $B = 0$ and the other is bubbly with $B > 0$. The local equilibria around the bubbleless steady state have indeterminacy of degree 1. The bubbly steady state is a saddle and the local equilibrium around this steady state is unique. The bubble can collapse as in Weil (1987) and there is a stochastic bubble equilibrium.

Of course the MW model is related to the multiple equilibria literature (e.g., Farmer (1999)). This literature typically has a unique steady state. The multiplicity is generated by the failure of the Blanchard-Khan condition in the sense that the number of predetermined variable is less than the number of stable eigenvalues for the linearized model around the steady state. One can broadly interpret that the bubble literature belongs to the multiple equilibria literature because there are typically multiple equilibria in the bubble literature, e.g., the Tirole (1985) model. Moreover, all pure bubble models (e.g., the monetary models of Samuleson (1958) and

Lagos et al (2017)) feature multiple equilibria: one is the monetary equilibrium and the other is the non-monetary (also called fundamental or autarky) equilibrium.

We view that the multiplicity of equilibria is a key strength of rational bubble theory. After all, the literature of rational bubbles is motivated by the apparent excessive fluctuation in asset prices relative to the measured fundamental. In a quantitative study, Miao, Wang and Xu (2015) show the framework developed by MW can successfully replicate some key moments of stock prices including their excessive volatility and the comovement with the real economy. We think HT's understanding and interpretation of the literature are misleading and narrow-minded. By contrast, MW try to be open to the different interpretations of a model without affecting its economic substance. The ultimate purpose of a theory or a model is to explain the real world economic phenomena. Simply judging a model's contribution by whether it has used HT's so called rational bubble term is absurd to us.

3 Conclusion

HT provide a definition of rational bubbles and then claim the MW model does not feature a rational bubble according to their definition. Their definition and arguments follow the standard literature and they do not provide any new results or new insights into our understandings of the bubble literature. In this report, we have argued that their definition of rational bubbles has limitations for several reasons. Some reasons have already been discussed by Tirole (1985). Their definition applies to some models, but not to others with financial or market frictions, e.g., the MW model in which stock price bubbles help generate dividends. The Bubble Characterization Lemma also does not apply to the MW model. In the MW model, the bubble component satisfies a different pricing equation than the traditional one, and thus the usual transversality condition fails to rule out the bubble component in the MW model. This equation is the same as that for the pure bubble asset like money so that the interpretation of the bubble component can be justified. The critiques by HT do not apply to the MW model. Many of their claims are baseless and reflect their misunderstandings and misinterpretations of the MW model in particular and the bubble literature in general.

References

Farmer, R. E. A. (1999). Macroeconomics of Self-Fulfilling Prophecies. 2nd ed. MIT Press.

Fostel, A., and J. Geanakoplos, 2008, "Leverage Cycles and the Anxious Economy," American Economic Review, 98(4), 1211–1244.

Garleanu, Nicolae and Lasse Heje Pedersen, 2011, Margin-based Asset Pricing and Deviations from the Law of One Price, Review of Financial Studies 24, 1980-2022.

Geanakopolos, John. 2003. "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium." In Advances in Economics and Econometrics: Theory and Applications. Econometric Society Monographs. Eighth

He, Hua, and David M. Models, 1995, Market Frictions and Consumption-based Asset Pricing, Journal of Political Economy 103, 94-117.

Lagos, R., G. Rocheteau, and R. Wright (2017). "Liquidity: A New Monetarist Perspective". Journal of Economic Literature 55.2, 371–440.

LeRoy, S. and J. Werner, 2001, Principles of Financial Economics, Cambridge University Press.

Miao, J. (2014). Economic Dynamics in Discrete Time. 2nd edition. Cambridge, MA: MIT Press.

Miao, J. and P. Wang (2018). "Asset Bubbles and Credit Constraints". American Economic Review 108.9, 2590–2628.

Montrucchio, L. (2004). "Cass Transversality Condition and Sequential Asset Bubbles". Economic Theory 24.3, 645–663.

Samuelson, P. A. (1958). "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money". Journal of Political Economy 66.6, 467–482. doi: 10.1086/258100.

Santos, M. S. and M. Woodford (1997). "Rational Asset Pricing Bubbles". Econometrica 65.1, 19–57. doi: 10.2307/2171812.

Tirole, J. (1985). "Asset Bubbles and Overlapping Generations". Econometrica 53.6, 1499–1528.

Weil, Philippe, 1987, Confidence and the Real Value of Money in an Overlapping Generations Economy, Quarterly Journal of Economics 102, 1-22.