Discussion of “Earnings Inequality and Other Determinants of Wealth Inequality” by Benhabib, Bisin, and Luo

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Models of wealth inequality

- Studying wealth inequality has become quite popular recently.
- Wealth inequality arises in heterogeneous-agent models.
- Literature has considered two types of uninsurable idiosyncratic income risks:

  Endowment risk Bewley (ECMA 1983), Huggett (JEDC 1993), Aiyagari (QJE 1994), Krusell & Smith (JPE 1998), Calvet (JET 2001), Angeletos & Calvet (JMathE 2005, JME 2006), Wang (JME 2007), ...

  Investment risk Krebs (QJE 2003), Cagetti & De Nardi (JPE 2006), Angeletos (RED 2007), Luttmer (QJE 2007, REStud 2011), Benhabib, Bisin, & Zhu (ECMA 2011, JET 2015, MD 2016), Toda (JET 2014), Toda & Walsh (JPE 2015), Aoki & Nirei (AEJ:Mac 2016), Gabaix, Lasry, Lions, & Moll (ECMA 2016), ...

Endowment risk alone is insufficient

- It has been known (numerically) at least since late 90's that endowment risk alone is insufficient to generate fat-tailed (Pareto) wealth distribution observed in data.
- Intuition: since labor income risk is transitory, there is no incentive to accumulate a lot of wealth.
- To get realistic wealth distribution, Krusell & Smith (1998) assume that discount factor is stochastic, and so is saving rate.

Random growth generates Pareto tails

- Incidentally, since late 90's random growth models have been used to explain Pareto tails (many papers by Gabaix, Reed, Benhabib, Bisin, & Zhu, Toda)
- Summary of some results:
  - Random growth ($S_t = G_t S_{t-1}$) with a lower reflective boundary generates power law (Gabaix, 1999)
  - Geometric Brownian motion with birth/death generates double Pareto distribution (Reed, 2001)
  - Also true in discrete time with arbitrary shocks (Toda, 2014)
  - In micro-founded OLG model with inheritance, investment risk generates power law (Benhabib, Bisin, & Zhu, 2011), using Kesten (1973) process

Simple model with investment risk

- Heterogeneous agents with log utility $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t$
- Linear investment technology with idiosyncratic stochastic return R_{t+1}
- Budget constraint: $w_{t+1} = R_{t+1}(w_t - c_t)$
- Optimal consumption rule is $c_t = (1-\beta)w_t$, hence $w_{t+1} = \beta R_{t+1} w_t$: random growth
- $\log w_{t+1} = \log w_t + X_{t+1}$, where $X_t = \log \beta R_t$. Can show that when agents die and are born with constant probability (perpetual youth model), then $\log w_t$ has exponential tails (Beare & Toda, WP), so w_t has Pareto tails
- Note: random discount factor β plays same role as random returns

  $\implies$ explains why Krusell & Smith (1998) needed random β

How about labor income risk?

- In general, hard to (analytically) solve models with labor income risk
- For tail behavior, only decision rule for large wealth is important, so assume an affine consumption rule $c = aw + b$
- Then wealth accumulation is of the form

$$w_{t+1} = A_{t+1}w_t + B_{t+1},$$

  which is known as Kesten (1973) process
- Authors show that in this case, stationary wealth distribution has a Pareto upper tail, which is the fatter tail of either
  1. labor income process itself, or
  2. an exponent determined solely by capital income risk
- Implication: labor income risk does not help to generate a fatter tail in wealth

Comments

- Nice negative result
- It would be even better if authors have a truly micro-founded model
- Suggestion: use CARA, because it goes well along with additive shocks (Calvet, 2001; Wang, 2007)

Bewley-Aiyagari economy with CARA utilities

- Agents maximize $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$, where $u(c) = \mathrm{e}^{-\gamma c}/\gamma$ is CARA
- Constant gross risk-free rate R (no capital risk), i.i.d. income y
- Only state variable is wealth w; can guess $V(w) = -\frac{1}{\gamma a}\mathrm{e}^{-\gamma(aw+b)}$ for some a, b
- By guess-and-verify, it turns out that consumption rule is $c = aw + b$, $a = 1 - 1/R$, and

$$b = \frac{1}{\gamma(1-R)} \log \beta R \, \mathrm{E}[\mathrm{e}^{-\gamma(1-1/R)y}]$$

Wealth distribution in Bewley-Aiyagari economies

- Wealth accumulation is $w_{t+1} = w_t - bR + y_{t+1}$, so wealth is random walk in levels (not logs)!
- No stationary distribution if agents infinitely lived; with constant birth/death, wealth distribution is Laplace (thin tailed)
- Obtain similar results if income is AR(1) with arbitrary shocks
- Theoretically explains why Bewley-Aiyagari models cannot generate fat-tailed wealth distribution