

# Referee Report on Necessity of Rational Asset Price Bubbles in Two-Sector Growth Economies

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## 1 Summary

The paper under review presents plausible economic models in which an equilibrium with rational asset price bubbles exists but equilibria with asset prices equal to fundamental values do not. These economies feature multiple sectors with faster economic growth than dividend growth. In our two-sector endogenous growth model, entrepreneurs have access to a production technology subject to idiosyncratic investment risk (tech sector) and trade a dividend-paying asset (land). When leverage is relaxed beyond a critical value, the unique trend stationary equilibrium exhibits a phase transition from the fundamental regime to the bubbly regime with growth, implying the inevitability of bubbles with loose financial conditions.

## 2 Comments

This is a theoretical study on the emergence conditions and properties of asset price bubbles with exogenous dividends (rents). While I find the topic interesting and would like to see more studies on this topic, I have a few reservations and concerns.

1. (**Relative contribution**) The authors motivate their study by discussing three shortcomings of the literature: First, the literature has almost exclusively focused on pure bubbles like fiat money, instead of bubbles attached to assets with dividends/rents. Second, existing models suffer from equilibrium indeterminacy. Third, bubbles are more likely to emerge when financial frictions are sufficiently tight. However, the authors fail to fairly assess the developments of the literature.

First, even the early seminal contribution by Tirole (1985, ECMA) has already studied bubbles attached to assets with dividends (or rents). He first assumes rents are constant over time without growth just like this submission (see further discussion below). He then considers the case where rents can grow as fast as the economy. Later, Miao, Wang, and Xu (2015, QE), Miao and Wang (2018, AER), and Jiang, Miao, and Zhang (2022, IER) also study models of

bubbles attached to assets with positive dividends/rents. Unfortunately, the authors fail to discuss or cite these papers, even though they are all published in top economic journals.

Second, one of the main attractive features of models with asset bubbles is that these models can generate multiple equilibria and hence can explain data using nonfundamental shocks (self-fulfilling beliefs) instead of fundamental shocks alone. I do not think this is a shortcoming. Rather, multiplicity and self-fulfilling beliefs are defining features of models of asset bubbles. By contrast, the paper under review claims that it provides a model of asset bubbles with a unique equilibrium. I am not convinced if this model makes intuitive sense or contributes to the literature in an interesting way. For example, why do not self-fulfilling beliefs play a role in this model? Can the bubble burst?

Third, while I find the result interesting that bubbles emerge under loose financial constraints, I am not quite convinced by this result. In the extreme case when financial constraints are sufficiently loose, the model would converge to the one without financial frictions so that a bubble can never emerge. Thus, I expect the paper's result might be that a bubble emerges in the intermediate range of financial constraints. In any case, more discussions in the paper are needed.

2. (**Analysis**) I am not convinced by one of the main result of the paper that an equilibrium with rational asset price bubbles exists but equilibria with asset prices equal to fundamental values do not. Let me use a simple asset pricing equation to illustrate the idea:

$$P_t = \frac{P_{t+1} + D}{R},$$

where dividends  $D$  are constant over time, while  $P_t$  grows at gross rate  $G > 1$ . Then we have

$$p_t = \frac{Gp_{t+1}}{R} + \frac{D}{RG^t},$$

where  $p_t = P_t/G^t$ . In the long-run trend stationary equilibrium  $p_t = p$  for all  $t$ , we have

$$p = \frac{G}{R}p. \tag{1}$$

We can see that there may be two types of equilibria: bubbly equilibrium in which  $p > 0$  and  $G = R$  and bubbleless equilibrium with  $p = 0$ .

The above analysis essentially summarizes the idea of Tirole (1985). The above equations and analysis also apply to the paper under review. By contrast, section 2 of the paper presents two examples in which the bubbleless equilibrium does not exist. I am not convinced by this result because they only demonstrate the “bubbleless equilibrium” with a particular trading strategy does not exist. In both the OLG model and infinite-horizon model of section 2, the authors only consider a specific trading strategy. For example, they assume that the old sells all of the asset and the young buys all of the asset in the OLG model. Is it possible that there

is no trade in the long-run trend stationary steady state? My understanding is that equation (1) is the same as that for the pure bubble case and hence the existence condition would be similar.

3. (**Modeling**) The model of the paper is not quite innovative as it is based on Kiyotaki and Moore model and Hirano and Yanagawa (2017). I find several additional features of the model are problematic or need further discussions.
  - (a) I find their emphasis of the two sectors misleading because there is actually no separate land/housing sector. I do not think it makes much sense to interpret the asset with exogenous rents as the land sector. At least, I would expect that there is production of different goods using factor resources in both sectors.
  - (b) I find the assumption of exogenous dividends/rents that do not grow or grow at a slower rate than the economy unrealistic or undesired. Tirole (1985) already notices this issue and also studies the case where dividends/rents grow as fast as the economy (see pp 1507-1509 of his paper). Moreover, rents/dividends should be endogenous. Modeling asset bubbles with endogenous rents would be much more interesting and challenging.
  - (c) I find the modeling of leverage constraint in the paper nonstandard. The leverage ratio is defined as value to cash/equity ratio. The leverage constraint is equivalent to a loan-to-value constraint. In the paper the budget constraint is

$$c_t + k_t + P_t x_t + b_t = z_{t-1} k_{t-1} + (P_t + D) x_{t-1} + R_{t-1} b_{t-1}.$$

The authors define the leverage constraint as

$$k_t \leq \lambda_t (k_t + P_t x_t + b_t).$$

I am not convinced by this constraint. Since borrowing means  $b_t < 0$  and investments are  $(k_t + P_t x_t)$ , I think the loan-to-value constraint is given by

$$-b_t \leq \lambda_t (k_t + P_t x_t).$$

Geneakoplos and his coauthors model leverage constraints in a different way with many financial assets.

- (d) Should there be a short-sales constraint on asset trading? This is a standard constraint to generate asset bubbles (Kocherlakota 1992).
- (e) I find the modeling of the idiosyncratic productivity shock  $z_t$  nonstandard. The authors assume that the date  $t$  output is given by  $z_{t-1} k_{t-1}$ , which is affected by the productivity shock in the last period  $t - 1$ . Why does not the current productivity affect current output? I can understand this timing assumption is critical to derive a nice solution in the paper. But I think the paper should provide convincing discussions on this assumption.

4. (**Literature**) As I already mentioned above, the paper does not provide a fair discussion on the related literature or even does not cite many related papers.