Land Bubbles Despite Non-Vanishing Rents*

Alexis Akira Toda†

October 25, 2025

Abstract

Skeptics of rational land or housing bubbles have claimed that bubbles do not exist on the basis of the supposed stationarity in the rent yield (rent to price ratio), as a bubble implies a vanishing rent yield. To counter this claim, we present a simple model in which the housing rent yield is constant but a land bubble inevitably emerges nevertheless. The confusion arises from not distinguishing the rent on the housing structure from the pure land rent.

Keywords: asset price bubble, bubble necessity, dividend-paying asset, dividend yield, land, rent.

JEL codes: D53, G12.

1 Introduction

Until around 2023, the large literature on rational bubbles (a situation in which the asset price exceeds its fundamental value defined by the present value of dividends in a rational equilibrium model) has been mostly restricted to the analysis of *pure bubbles*, meaning that an intrinsically worthless asset like fiat money is traded at a positive price.[1] However, as Hirano and Toda (2024, §4.7) argue, pure

*I thank Nobuhiro Kiyotaki for suggesting the importance of separating rent accruing to land and structure. I acknowledge financial support from Japan Center for Economic Research; Japan Securities Scholarship Foundation; and the Joint Usage/Research Center, Institute of Economic Research, Hitotsubashi University (Grant ID: IERPK2515).

†Department of Economics, Emory University, alexis.akira.toda@emory.edu.

[1] See Martin and Ventura (2018) and Hirano and Toda (2024) for reviews of the rational bubble literature. Examples of rational bubbles on dividend-paying assets include Wilson (1981, §7), Tirole (1985, Proposition 1(c)), Olivier (2000), Allen et al. (2017, §2), and Bosi et al. (2018) as well as their related papers. Pham and Toda (2025) point out that the original analysis in

bubble models have some limitations including lack of realism, equilibrium indeterminacy, and the inability to connect to the econometric literature that uses the price-dividend ratio. These limitations have prompted skepticism against rational bubble models.

Once we consider bubbles attached to dividend-paying assets, the theoretical implications fundamentally change. Since 2023, Tomohiro Hirano and I have discussed elsewhere (Hirano and Toda, 2023, 2024, 2025a,c) that under some conditions, rational bubbles on dividend-paying assets inevitably emerge (*bubble necessity*) whenever the dividend growth rate exceeds the natural interest rate (the interest rate that prevails without the asset). In these models, an implication of bubbles is that the dividend yield (dividend to price ratio) vanishes over time. Some people argue that this is counterfactual because the rent to price ratio is supposed to be stationary.[2]

The purpose of this paper is to counter the claim of some empirical researchers who dismiss the existence of rational bubbles in land or housing just on the basis of the supposed stationarity in the price-rent ratio. We present a simple (minimal) model in which a land bubble necessarily arises, even though housing rent to price ratio is stationary (in fact, constant). The key is that we need to distinguish rents accruing to the housing structure and land. It could be the case that the housing rent consists mostly of the rent on housing structure. We cannot exclude the possibility of a land bubble without separating the pure land rent.

2 Model

Consider a two-period overlapping generations model with two commodities, consumption good and housing service.

Preferences and endowments The utility function of generation t is Cobb-Douglas,

$$U(c_t^y, c_{t+1}^o, h_t) = \beta_y \log c_t^y + \beta_o \log c_{t+1}^o + \beta_h \log h_t, \tag{1}$$

Tirole (1985) was incorrect. Olivier (2000) considers a model in which the law of one price is violated (assets with identical payoffs but different vintages are traded at different prices). The example in Allen et al. (2017) is essentially the same as Wilson (1981).

[2] For instance, an anonymous reviewer of one of my papers (not this paper) claimed "[In the model] price-to-rent ratios are non-stationary and grow without bound. This seems patently at odds with data", although without presenting any evidence. Hirano and Toda (2025c, Fig. 1) present evidence of declining land share and Bäcker-Peral et al. (2025, Fig. 1) present evidence of declining housing yield.

where c_t^y, c_{t+1}^o are consumption when young and old, h_t is housing service, and $\beta_y, \beta_o, \beta_h > 0$ are parameters with $\beta_y + \beta_o + \beta_h = 1$. The interpretation of (1) is that agents consume housing only when transitioning from young to old. This assumption is not essential but is convenient for reducing the dimension of the equilibrium system. At time t, the young are endowed with $e_t > 0$ units of the consumption good and the old none. There is a unit supply of land initially owned by the old, which is durable and non-reproducible.

Technology There is a representative firm that produces housing structure using construction material denoted by $M > 0$ and land denoted by $X > 0$. Let $F(M, X)$ be the production function of housing. For simplicity, assume that construction material is the same as the consumption good (whose price is normalized to 1) and housing is perishable and fully depreciates between periods. (It is straightforward to introduce partial depreciation of housing structure.) At time t, let r_t be the price of housing (housing rent), P_t the land price, and D_t the dividend of land (land rent). The firm's problem is to maximize the profit

$$r_t F(M, X) - M - D_t X. \tag{2}$$

We impose the following conditions.

Assumption 1. *The production function $F : \mathbb{R}^2_{++} \to (0, \infty)$ is (i) homogeneous of degree 1, (ii) continuously differentiable with $F_M > 0$, $F_X > 0$, (iii) concave, and (iv) $\lim_{M \to 0} (F_X/F_M)(M, 1) = 0$.*

Conditions (i)–(iii) imply that F is a standard neoclassical production function. Condition (iv) is a sort of Inada condition. There are many functions satisfying Assumption 1, for instance the constant elasticity of substitution (CES) production function

$$F(M, X) = \begin{cases} A \left(\alpha M^{1-1/\sigma} + (1-\alpha) X^{1-1/\sigma} \right)^{\frac{1}{1-1/\sigma}} & \text{if } 0 < \sigma \neq 1, \\ A M^{\alpha} X^{1-\alpha} & \text{if } \sigma = 1, \end{cases} \tag{3}$$

where $\sigma > 0$ is the elasticity of substitution between the construction material and land and $A > 0$, $\alpha \in (0, 1)$ are parameters.

Budget constraints Letting x_t be the demand for land by the young, generation t's problem is to maximize utility (1) subject to the budget constraints

$$\text{Young:} \qquad c_t^y + P_t x_t + r_t h_t = e_t, \tag{4a}$$

$$\text{Old:} \qquad c_{t+1}^o = (P_{t+1} + D_{t+1}) x_t. \tag{4b}$$

Thus, the young spend income on consumption, housing rent, and land purchase, while the old finance their consumption by selling land and receiving dividends. Letting

$$R_t := \frac{P_{t+1} + D_{t+1}}{P_t} \tag{5}$$

be the implied gross risk-free rate, we may combine the two budget constraints (4) into one as

$$c_t^y + \frac{c_{t+1}^o}{R_t} + r_t h_t = e_t. \tag{6}$$

Equilibrium The definition of an equilibrium is standard.

Definition 1. A *rational expectations equilibrium* consists of sequences of prices $\{(r_t, P_t, D_t)\}_{t=0}^{\infty}$ and quantities $\{(c_t^y, c_t^o, h_t, x_t, M_t, X_t)\}_{t=0}^{\infty}$ such that for each t, (i) (Utility maximization) Generation t maximizes utility (1) subject to the budget constraints (4); (ii) (Profit maximization) the firm maximizes the profit (2); (iii) (Commodity market clearing) $c_t^y + c_t^o + M_t = e_t$; (iv) (Housing market clearing) $h_t = F(M_t, X_t)$; and (v) (Land market clearing) $x_t = 1 = X_t$.

3 Equilibrium land price

We solve for the equilibrium. Because the utility function (1) is Cobb-Douglas, maximizing it subject to the combined budget constraint (6) yields the optimal expenditures

$$(c_t^y, c_{t+1}^o / R_t, r_t h_t) = (\beta_y e_t, \beta_o e_t, \beta_h e_t). \tag{7}$$

Using the budget constraint of the young (4a) and the land market clearing condition $x_t = 1$, we obtain the land price

$$P_t = P_t x_t = \beta_o e_t. \tag{8}$$

The following proposition is an immediate implication of these derivations.

Proposition 1. *Aggregate rent to price ratio is constant at* $r_t h_t / P_t = \beta_h / \beta_o$.

Profit maximization and land market clearing $X_t = 1$ imply

$$1 = r_t F_M(M_t, 1), \tag{9a}$$

$$D_t = r_t F_X(M_t, 1). \tag{9b}$$

Using the expenditure formula (7), (9a), and housing market clearing $h_t = F(M_t, 1)$, we obtain the equilibrium condition

$$F(M_t, 1) = h_t = \frac{\beta_h e_t}{r_t} = \beta_h e_t F_M(M_t, 1) \iff \frac{F}{F_M}(M_t, 1) = \beta_h e_t. \tag{10}$$

Using the homogeneity of F, we may rewrite the function F/F_M in (10) as

$$\frac{F}{F_M}(M, 1) = \frac{MF_M + F_X}{F_M}(M, 1) = M + \frac{F_X}{F_M}(M, 1). \tag{11}$$

The homogeneity and concavity of F implies F_X/F_M is increasing in M. Hence, by (11) and Assumption 1(iv), the function $M \mapsto (F/F_M)(M, 1)$ is strictly increasing and has range $(0, \infty)$. Consequently, for any $e_t > 0$, there exists a unique $M_t > 0$ satisfying (10).

Finally, we check commodity market clearing. Using the budget constraint of the old (4b) with $x_t = 1$, we obtain

$$c_t^y + c_t^o + M_t = c_t^y + (P_t + D_t) + M_t. \tag{12}$$

Noting that F is homogeneous of degree 1 and using (9), we obtain

$$h_t = F(M_t, 1) = F_M(M_t, 1)M_t + F_X(M_t, 1) = \frac{1}{r_t}(M_t + D_t). \tag{13}$$

Combining (7), (8), (12), and (13), we obtain

$$\begin{aligned} c_t^y + c_t^o + M_t &= c_t^y + P_t + r_t h_t \\ &= \beta_y e_t + \beta_o e_t + \beta_h e_t = e_t, \end{aligned}$$

so the commodity market clears. We thus obtain the following proposition.

Proposition 2. *There exists a unique equilibrium, which is characterized by (7)–(10). There is a land bubble if and only if*

$$\sum_{t=1}^{\infty} \frac{F_X}{F}(M_t, 1) < \infty, \tag{14}$$

where M_t uniquely solves (10).

We refer the reader to Hirano and Toda (2024, §2) and Hirano and Toda (2025b) for the formal definition of rational bubbles.

For instance, suppose F is the CES production function (3). Following the empirical evidence, set $\sigma > 1$.[3] A straightforward calculation shows

$$\frac{F_M}{F}(M,1) = \frac{\alpha M^{-1/\sigma}}{\alpha M^{1-1/\sigma} + 1 - \alpha} \sim M^{-1}, \tag{15a}$$

$$\frac{F_X}{F}(M,1) = \frac{1-\alpha}{\alpha M^{1-1/\sigma} + 1 - \alpha} \sim \frac{1-\alpha}{\alpha} M^{1/\sigma - 1} \tag{15b}$$

as $M \to \infty$. Therefore we obtain the following characterization of land bubbles.

Proposition 3. *If the production function takes the CES form* (3) *with elasticity of substitution $\sigma > 1$, there is a land bubble if and only if*

$$\sum_{t=1}^{\infty} e_t^{1/\sigma - 1} < \infty. \tag{16}$$

If the economy grows at rate $G > 1$, so $e_t \sim G^t$, then (16) is satisfied. Therefore, land bubbles inevitably emerge with economic growth. As Proposition 1 shows, if one is not careful and does not distinguish housing rent (price of housing service) from land rent (price of land use), one may incorrectly conclude that bubbles do not exist. More empirical work is necessary to determine the existence or nonexistence of land bubbles.

A Proofs

A.1 Proof of Proposition 2

It suffices to show that (14) is necessary and sufficient for a land bubble. Using (9), the land dividend is given by $D_t = (F_X/F_M)(M_t, 1)$. Using (8) and (10), the dividend yield is given by

$$\frac{D_t}{P_t} = \frac{F_X}{F_M}(M_t, 1)\frac{\beta_h}{\beta_o}\frac{F_M}{F}(M_t, 1) = \frac{\beta_h}{\beta_o}\frac{F_X}{F}(M_t, 1).$$

[3] Epple, Gordon, and Sieg (2010) find that the elasticity of substitution between land and non-land factors for producing housing is 1.16 for residential properties and 1.39 for commercial properties in Allegheny County, Pennsylvania. Ahlfeldt and McMillen (2018) argue that the estimation approach of Epple et al. (2010) is less susceptible to measurement error than older estimates, which are likely biased downwards. They find that the elasticity of substitution is around 1.25 for Chicago.

The claim follows from the Bubble Characterization Lemma (Proposition 7 of Montrucchio (2004) or Lemma 1 of Hirano and Toda (2025a)): see Lemma B.1. $\square$

A.2 Proof of Proposition 3

We first show sufficiency. If (16) holds, since $\sigma > 1$, we must have $e_t \to \infty$ as $t \to \infty$. Using (10) and (15a), it follows that $M_t \to \infty$. For any $\epsilon \in (0,1)$, we can take $M(\epsilon) > 0$ such that $M > M(\epsilon)$ implies

$$\frac{\alpha M^{1-1/\sigma}}{\alpha M^{1-1/\sigma} + 1 - \alpha} \in (1-\epsilon, 1).$$

Specifically, we may set

$$M(\epsilon) = \left[\frac{(1-\epsilon)(1-\alpha)}{\epsilon\alpha}\right]^{\frac{1}{1-1/\sigma}}. \tag{17}$$

Take $T > 0$ such that $M_t > M(\epsilon)$ for $t > T$. Then (10) and (9a) imply

$$\frac{M_t}{\beta_h e_t} = M_t \frac{F_M}{F}(M_t, 1) = \frac{\alpha M_t^{1-1/\sigma}}{\alpha M_t^{1-1/\sigma} + 1 - \alpha} \in (1-\epsilon, 1) \tag{18}$$

for $t > T$. By (15b), we obtain

$$\frac{F_X}{F}(M_t, 1) \leq \frac{1-\alpha}{\alpha} M_t^{1/\sigma - 1} \leq \frac{1-\alpha}{\alpha}[(1-\epsilon)\beta_h e_t]^{1/\sigma-1},$$

which is summable by (16). Therefore there is a land bubble by Proposition 2.

We next show necessity. If there is a land bubble, by Proposition 2, (14) holds. Using (15b) and $\sigma > 1$, it must be $M_t \to \infty$ as $t \to \infty$. Take $\epsilon \in (0,1)$ and let $M(\epsilon)$ be as in (17). Take $T > 0$ such that $M_t > M(\epsilon)$ for $t > T$. Then (15b) and (18) imply

$$\begin{aligned}\frac{F_X}{F}(M_t, 1) &= \frac{1-\alpha}{\alpha} \frac{\alpha M_t^{1-1/\sigma}}{\alpha M_t^{1-1/\sigma} + 1 - \alpha} M_t^{1/\sigma - 1} \\ &\geq \frac{1-\alpha}{\alpha}(1-\epsilon) M_t^{1/\sigma-1} \geq \frac{1-\alpha}{\alpha}(1-\epsilon)(\beta_h e_t)^{1/\sigma - 1}\end{aligned}$$

for $t > T$. Summing over t and using (14), we obtain (16). $\square$

References

Ahlfeldt, G. M. and D. P. McMillen (2018). “Tall buildings and land values: Height and construction cost elasticities in Chicago, 1870–2010”. *Review of Economics and Statistics* 100.5, 861–875. DOI: 10.1162/rest_a_00734.

Allen, F., G. Barlevy, and D. Gale (2017). *On Interest Rate Policy and Asset Bubbles*. Working Paper 2017-16. Federal Reserve Bank of Chicago. URL: https://www.econstor.eu/handle/10419/200566.

Bäcker-Peral, V., J. Hazell, and A. Mian (2025). “Dynamics of the long term housing yield: Evidence from natural experiments”. URL: https://jadhazell.github.io/website/UK_Duration.pdf.

Bosi, S., T. Ha-Huy, C. Le Van, C.-T. Pham, and N.-S. Pham (2018). “Financial bubbles and capital accumulation in altruistic economies”. *Journal of Mathematical Economics* 75, 125–139. DOI: 10.1016/j.jmateco.2018.01.003.

Epple, D., B. Gordon, and H. Sieg (2010). “A new approach to estimating the production function for housing”. *American Economic Review* 100.3, 905–924. DOI: 10.1257/aer.100.3.905.

Hirano, T. and A. A. Toda (2023). “Housing bubbles with phase transitions”. arXiv: 2303.11365 [econ.TH].

Hirano, T. and A. A. Toda (2024). “Bubble economics”. *Journal of Mathematical Economics* 111, 102944. DOI: 10.1016/j.jmateco.2024.102944.

Hirano, T. and A. A. Toda (2025a). “Bubble necessity theorem”. *Journal of Political Economy* 133.1, 111–145. DOI: 10.1086/732528.

Hirano, T. and A. A. Toda (2025b). “Toward bubble clarity”. *Econ Journal Watch* 22.1, 1–17. URL: https://econjwatch.org/1384.

Hirano, T. and A. A. Toda (2025c). “Unbalanced growth and land overvaluation”. *Proceedings of the National Academy of Sciences* 122.14, e2423295122. DOI: 10.1073/pnas.2423295122.

Martin, A. and J. Ventura (2018). “The macroeconomics of rational bubbles: A user’s guide”. *Annual Review of Economics* 10, 505–539. DOI: 10.1146/annurev-economics-080217-053534.

Montrucchio, L. (2004). “Cass transversality condition and sequential asset bubbles”. *Economic Theory* 24.3, 645–663. DOI: 10.1007/s00199-004-0502-8.

Olivier, J. (2000). “Growth-enhancing bubbles”. *International Economic Review* 41.1, 133–152. DOI: 10.1111/1468-2354.00058.

Pham, N.-S. and A. A. Toda (2025). "Rational bubbles on dividend-paying assets: A comment on Tirole (1985)". Revise and resubmit at *Econometrica*. arXiv: 2507.12477 [econ.TH].

Tirole, J. (1985). "Asset bubbles and overlapping generations". *Econometrica* 53.6, 1499–1528. DOI: 10.2307/1913232.

Wilson, C. A. (1981). "Equilibrium in dynamic models with an infinity of agents". *Journal of Economic Theory* 24.1, 95–111. DOI: 10.1016/0022-0531(81)90066-1.

Online Appendix (Not for publication)

B Fundamental results on asset price bubbles

This appendix reviews the fundamental results on asset price bubbles (Santos and Woodford, 1997; Hirano and Toda, 2024, 2025a,b).

Consider a deterministic infinite-horizon economy with time indexed by $t = 0, 1, \dots$. Let $q_t > 0$ be the date-0 price (Arrow-Debreu price) of the date-t good with normalization $q_0 = 1$. Suppose an asset pays dividend D_t and trades at price $P_t > 0$ at time t. The absence of arbitrage implies

$$q_t P_t = q_{t+1}(P_{t+1} + D_{t+1}).$$

Iterating over t, for any $T > t$ we obtain

$$q_t P_t = \sum_{s=t+1}^{T} q_s D_s + q_T P_T. \tag{19}$$

Since $q_t > 0$, $P_t \geq 0$, and $D_t \geq 0$ for all t, the partial sum $\left\{ \sum_{s=t+1}^{T} q_s D_s \right\}$ is increasing in T and bounded above by $q_t P_t$, so it is convergent. Therefore, letting $T \to \infty$ in (19) and dividing both sides by $q_t > 0$, we obtain

$$P_t = \frac{1}{q_t} \sum_{s=t+1}^{\infty} q_s D_s + \frac{1}{q_t} \lim_{T \to \infty} q_T P_T, \tag{20}$$

where

$$V_t := \frac{1}{q_t} \sum_{s=t+1}^{\infty} q_s D_s \tag{21}$$

is the fundamental value (present discounted value of dividends). Comparing (20) and (21), we have $P_t \geq V_t$ for all t. We say that the asset price contains a *bubble* at time t if $P_t > V_t$, or in other words, the asset price P_t exceeds its fundamental value V_t defined by the present value of dividends. Note that $P_t = V_t$ for all t if and only if

$$\lim_{T \to \infty} q_T P_T = 0. \tag{22}$$

We refer to (22) as the *no-bubble* condition.

The following Bubble Characterization Lemma plays a fundamental role in determining the existence of bubbles.

Lemma B.1 (Bubble Characterization, Montrucchio, 2004, Proposition 7)**.** *In any equilibrium, the asset price exhibits a bubble if and only if*

$$\sum_{t=0}^{\infty} \frac{D_t}{P_t} < \infty. \tag{23}$$

See Hirano and Toda (2025a, Lemma 1) for a simple proof of Lemma B.1. Lemma B.1 states that a bubble emerges if and only if the dividend yield D_t/P_t converges to zero sufficiently fast or the price-dividend ratio P_t/D_t grows sufficiently fast.

References

Hirano, T. and A. A. Toda (2024). "Bubble economics". *Journal of Mathematical Economics* 111, 102944. DOI: 10.1016/j.jmateco.2024.102944.

Hirano, T. and A. A. Toda (2025a). "Bubble necessity theorem". *Journal of Political Economy* 133.1, 111–145. DOI: 10.1086/732528.

Hirano, T. and A. A. Toda (2025b). "Toward bubble clarity". *Econ Journal Watch* 22.1, 1–17. URL: https://econjwatch.org/1384.

Montrucchio, L. (2004). "Cass transversality condition and sequential asset bubbles". *Economic Theory* 24.3, 645–663. DOI: 10.1007/s00199-004-0502-8.

Santos, M. S. and M. Woodford (1997). "Rational asset pricing bubbles". *Econometrica* 65.1, 19–57. DOI: 10.2307/2171812.