## Referee Report on JET-D-19-00061 by Toda and Stachurski

This is a useful and concise paper that contextualizes a recent literature on fat tailed income and wealth distributions, and provides a theorem showing why standard Aiyagari models cannot jointly match the income and wealth distributions in the data.

A number of recent papers have noted, mostly through simulations, that in standard Aiyagari models the right tail of income distribution is not thinner than that of wealth, but it is clearly so in the data. (see for example some references cited at the end of the report that note this difficulty). This paper provides a concise, clear and useful proof, an "impossibility result", showing that in the standard Aiyagari type models the is inescapable.

To address this empirical difficulty of the Aiyagari models a number of papers have introduced additional features that depart from the standard Aiyagari model, like stochastic returns, stochastic or heterogenous discount factors, non-homotheticities where savings rates or bequests increase in wealth, rates of return that increase in wealth, or alternatively they have resorted to birth and death or perpetual youth models where the very rich turn out to be unrealistically old.

To put this paper in context, the literature that introduced stochastic returns used results due to Kesten for linear models that establish the possibility of wealth having a fatter right tail than income, and provided a characterization of the power law wealth right tail in terms of the stochastic properties of the rate of return process. While this may provide the basic intuition for fatter right tails for the wealth distribution compared to right tails of income distribution, Aiyagari models with borrowing constraints are non-linear with non-linear policy functions, and Kesten type results do not immediaely apply. However with under homothetic preferences Aiyagari models are asymptotically linear, and generalizations of Kesten due to Mirek to non-linear contexts, cited in the paper, allow the Kesten results to prevail in Aiyagari models augmented with stochastic returns under appropriate regularity conditions.

This literature however has focused on "possibility results" that generate realistic fat tails of wealth. It only noted a version of the "impossibility results" (that wealth right tails will not differ from income tails in standard Aiyagari models) for linear models based on the paper by Grey that extends Kesten. The contribution of this paper is to extend Grey so it applies to standard the Aiyagari models that are only asymptotically linear

This is quite a valuable contribution and should generate many citations, as basic Aiyagari models are so widely used in economics. The proof extending Grey to non-linear contexts is elegant and simple (Theorem 2.3); it clearly spells out the general conditions needed for the result, and can be immediately applied to various types Aiyagari models: see examples 2, 3 and 4 in the paper. The examples also illustrate why stochastic returns, while not sufficient, may in fact be necessary to jointly match wealth and income distributions, as in Benhabib, Bisin and Luo, (AER forthcoming, 2019). References

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