Response to Miao and Wang

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Abstract

We submitted "Rational Bubbles: A Clarification" (Hirano and Toda, 2024) to *American Economic Review* as a comment to Miao and Wang (2018). Miao and Wang were assigned as referees and provided a report.[1] This document responds to their claim.

In what follows, we refer to Miao and Wang (2018) as MW18, the comment Hirano and Toda (2024) as HT, and Miao and Wang's response as MW24.

We have nothing to respond until the sentence on page 1 ending with "The proof is trivial".

On page 2, MW24 claim "MW already point out that the usual transversality condition holds in their model". However, this claim is misleading for two reasons. First, MW18 is unclear about the distinction between the "transversality condition for asset pricing" and the "transversality condition for optimality" (see Footnote 1 and Appendix C of HT), which are completely different. Second, the argument cited merely reiterates Section 1 of MW18, which serves to build intuition but is unrelated to the actual MW18 model.

The paragraph at the bottom of page 2 starting with "By contrast," merely reiterates MW18's view that the term B in $QK + B$ can be "interpreted" as a bubble. This is not an issue: HT acknowledge that there are diverse approaches to asset price bubbles. The point of HT is to prove that the "bubble" in MW18 is different from the classical "rational bubble" defined in Miao (2014, §13.6) and §2.1 in HT, despite the fact that MW18 muddied the distinction.

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[1] https://alexisakira.github.io/files/AER-2024-1053_Ref1.pdf

In the first half of the section titled "Definition of Rational Bubbles", MW24 seem to claim that the definition is ambiguous because dividends in the MW18 model are endogenous. This point is irrelevant because the standard definition of rational bubbles in §2.1 of HT is "let us call the present value of dividends the fundamental value; let us call the difference between the price and fundamental value the bubble". This definition does not ask where the dividends come from. Whether dividends are exogenous or endogenous is immaterial.

In the second half of the section titled "Definition of Rational Bubbles", MW24 claim that the proof of HT does not apply because "in many models with market frictions, the no-arbitrage pricing equation (7) may not hold" (bottom of page 4). The same claim is repeated in the section titled "Bubble Characterization Lemma". Here it suffices to point out that in the MW18 model, the marginal buyer of the stock is the representative household facing no financial frictions, and hence the no-arbitrage condition does hold. Therefore, MW24's claim is false.

In the section titled "Interpretation of B_t in (6)", MW24 repeat the argument of the interpretation. Again the point is irrelevant, because the issue raised by HT is whether the stock price (P) equals the fundamental value (V) defined by the present value of dividends (D) in the MW18 model. It is a mathematical question, not philosophical. MW24 conclude this section with the question "For consistency mentioned by HT, why do they say one is a rational bubble, but the other is not? Do they have double standards?" The reason why we say one is a rational bubble, while the other is not, is that "rational bubble" is a scientific term that has been defined and used for decades. (Recall the titles "Rational Asset Pricing Bubbles" of the seminal paper of Santos and Woodford (1997) or "The Macroeconomics of Rational Bubbles: A User's Guide" of the influential survey of Martin and Ventura (2018).) Scientists should not abuse well-established scientific terms. If MW18 would like to call their model bubble (without "rational"), we have no objections, as long as their definition and its distinction from the standard "rational bubble" are clear. The problem is that MW18 muddy the definition and misrepresent their scientific results.

In the section titled "Conclusion", MW24 state "The Bubble Characterization Lemma also does not apply to the MW model" without providing any supporting argument. It seems to contradict their own acknowledgment "The proof is trivial" at the bottom of page 1. As we have argued above, the lemma indeed applies to the MW18 model. MW24 conclude their report with the sentence "The critiques by HT do not apply to the MW model. Many of their claims are baseless and reflect their misunderstandings and misinterpretations of the MW model in particular

and the bubble literature in general". Their report is filled with strong adjectives such as "baseless", "dogmatic", "narrow", etc. We are disappointed that Miao and Wang chose to muddy the issues and appeal to personal attacks instead of engaging in a constructive scientific debate by directly responding to our mathematical argument.

References

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