Do You Save More or Less in Response to Bad News? A New Identification of the Elasticity of Intertemporal Substitution

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Background

- Elasticity of intertemporal substitution (EIS) is a key object that determines
  - Effects of monetary policy in New Keynesian models,
  - Effects of fiscal policy,
  - Consumption/saving decision in life cycle models.

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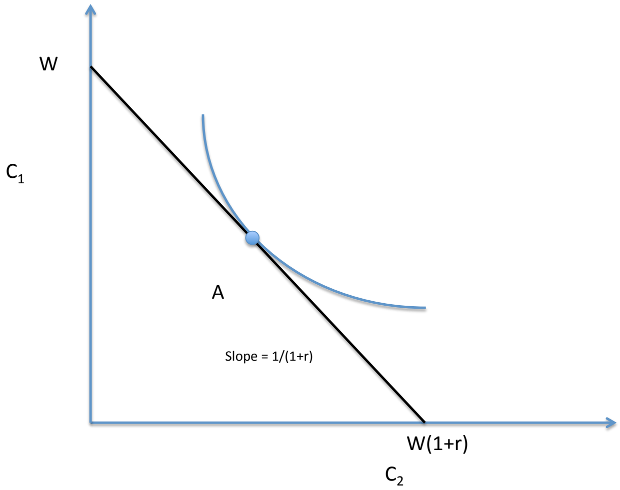
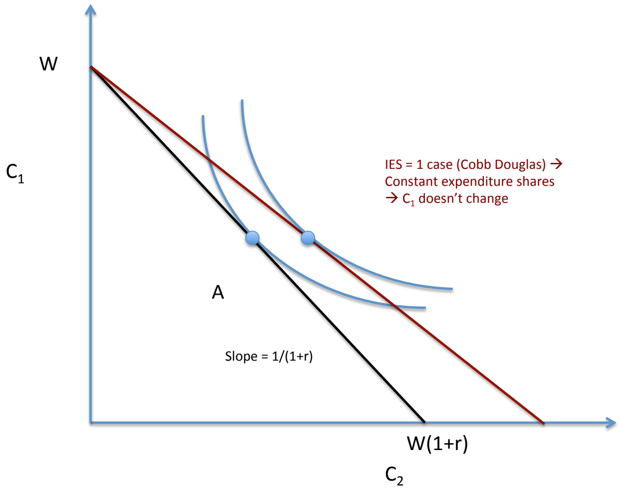
- Elasticity of intertemporal substitution (EIS) is a key object that determines
    - Effects of monetary policy in New Keynesian models,
    - Effects of fiscal policy,
    - Consumption/saving decision in life cycle models.
- Both authors work in the interdisciplinary (= gray) area between finance and macro and noticed that
    - Most macro papers assume $\mathrm{EIS} < 1$, typically around 0.5-0.8
    - Most finance papers assume $\mathrm{EIS} > 1$, typically around 1.5-2.
- Puzzling, because EIS should not depend on field.

Some econ 1 pictures...



Some econ 1 pictures...



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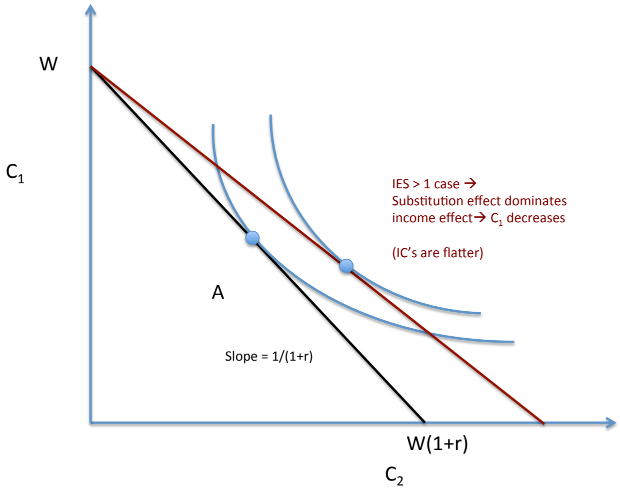

Why is EIS different across fields?

- Vast majority of empirical estimates are $\mathrm{EIS} < 1$ (more on this later). Probably macro literature follows these findings.
- Macro-finance ($\approx$ consumption-based capital asset pricing) literature (*e.g.*, Bansal & Yaron (2004)) assume $\mathrm{EIS} > 1$, because then can explain
  1. high equity premium,
  2. low & stable risk-free rate,
  3. pro-cyclical price-dividend ratio,
  4. negative price of risk for variance,
  5. ...

  which are much harder to explain with $\mathrm{EIS} < 1$.

Contribution

1. Define EIS for general recursive preferences.
2. Derive sharp comparative statics result between saving rate and "bad news" about future (any exogenous change that decreases continuation value):

   $\text{EIS} < 1$ Bad news $\implies$ savings $\uparrow$
   $\text{EIS} > 1$ Bad news $\implies$ savings $\downarrow$

   hence there is precautionary savings if and only if $\text{EIS} < 1$.
3. Hence our results provide a new way to test whether $\text{EIS} \gtrless 1$.

Conventional definition and estimation strategy

- Usual definition of EIS: elasticity of expected consumption growth on exogenous increase in risk-free rate.

$$\psi = \frac{\partial \operatorname{E}_t[\log c_{t+1}/c_t]}{\partial \log R_{f,t}},$$

where c_t: consumption, $R_{f,t}$: risk-free rate.

- Virtually all papers use consumption Euler equation to derive testable implication for response of consumption growth on risk-free rate.
- For identification, can rely on instruments or natural experiments (*e.g.*, tax increase).

What can go wrong

- Usually, we estimate the EIS via an IV regression of

$$\Delta c_{t+1} = \hat{\psi} r_{t+1} + u_{t+1}$$

with a predetermined instrument z_t.

- My JMP: Euler equation with representative agent, E-Z, preferences and i.i.d shocks:

$$E_t[\Delta c_{t+1}] = \psi E_t[r_{t+1}] \quad ,$$

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- Usually, we estimate the EIS via an IV regression of

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  with a predetermined instrument z_t.
- My JMP: Euler equation with incomplete markets, E-Z, preferences and non-i.i.d shocks:

$$E_t[\Delta c_{t+1}] = \psi E_t[r_{t+1}] + (\psi - 1)\vartheta_t + (\psi - 1)\,\mathrm{E}_t[\nu^*_{t+1}],$$

  additional terms: "certainty equivalents" over higher moments of aggregate and idiosyncratic shocks, respectively
- $\operatorname{plim}_{T\to\infty} \hat{\psi} = \psi + (\psi - 1)\frac{\mathrm{Cov}[z_t, \vartheta_t + \mathrm{E}_t[\nu^*_{t+1}]]}{\mathrm{Cov}[z_t, r_{t+1}]}$.
- Data suggest all three terms countercyclical–rising in recessions. GE considerations $\Rightarrow$ $E_t[r_{t+1}]$ rises for a reason...

EIS estimates

- Using aggregate consumption (representative agent), Hall (1988) gets at most 0.1. Mulligan (2002): around 1.5
- EIS not estimable from aggregate data if markets are incomplete. Hence many use micro data (Mankiw & Zeldes 1991, Attanasio & Weber 1993, Beaudry & van Wincoop 1996, Vissing-Jørgensen 2002, etc.). Estimates larger (0.3–1), but often less than 1.
- Gruber (2013) identifies EIS via cross-sectional and time-series variation in tax rates, and obtain 2.
- Cashin & Ueyama (forth.) VAT increase in Japan: 0.21.
- Best et al. (2015): interest rate notches in UK $\Rightarrow$ 0.05-0.25.
- Havranek et.al. (2015) review 2,735 estimates from 169 published studies, majority of which < 1, but SE $= 1.4$

Model

Single investor solves optimal consumption-portfolio problem.

- Time: $t = 0, 1, \dots, T$.
- Assets: $j = 1, \dots, J$. $R^j_{t+1} \geq 0$: gross return on asset j between time t and $t+1$.
- $\theta_t = (\theta^1_t, \dots, \theta^J_t)$: portfolio (vector of fraction of wealth invested in each asset, so $\sum_j \theta^j = 1$). $\theta^j > 0$ (< 0): long (short) position in asset j. Portfolio return:

$$R_{t+1}(\theta) := \sum_{j=1}^{J} R^j_{t+1} \theta^j.$$

- Portfolio constraint: $\theta_t \in \Theta_t \subset \mathbb{R}^J$.

Preferences

- Investor has general recursive preferences

$$U_t = f_t(c_t, \mathcal{M}_t(U_{t+1})),$$

where

  - U_t: continuation value at t,
  - $f_t(\cdot, \cdot)$: aggregator,
  - $\mathcal{M}_t(\cdot)$: certainty equivalent.

- Example: if

$$f(c, v) = ((1-\beta)c^{1-1/\psi} + \beta v^{1-1/\psi})^{\frac{1}{1-1/\psi}},$$

then CES aggregator with discount factor β and EIS ψ.

Assumptions

1. (CRRA) Certainty equivalent $\mathcal{M}_t$ is CRRA, so

$$\mathcal{M}_t(U) = \begin{cases} \mathrm{E}_t[U^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}, & (\gamma_t \neq 1) \\ \exp\left(\mathrm{E}_t[\log U]\right), & (\gamma_t = 1) \end{cases}$$

   where γ_t is relative risk aversion.
2. (Homotheticity) Aggregator $f_t : \mathbb{R}^2_+ \to \mathbb{R}_+$ is continuous, increasing, quasi-concave, & homogeneous of degree 1. Terminal utility is $u_T(c_T) = b_T c_T$ with $b_T > 0$.
3. Portfolio constraint Θ_t is nonempty, compact, & convex.

Note: $\gamma_t, f_t, b_T, \Theta_t$ can be time & state dependent.

Optimal portfolio problem

- w: wealth, $V_t(w)$: value function at t.
- Budget constraint

$$w_{t+1} = R_{t+1}(\theta_t)(w_t - c_t),$$

where θ_t: portfolio, c_t: consumption,
$R_{t+1}(\theta) = \sum_{j=1}^{J} R^j_{t+1}\theta^j$: gross portfolio return.

- Bellman equation:

$$V_t(w) = \max_{\substack{0 \le c \le w \\ \theta \in \Theta_t}} f_t\left(c, \mathrm{E}_t[V_{t+1}(R_{t+1}(\theta)(w-c))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right).$$

Solution

Lemma

1. *Value function is* $V_t(w) = b_t w$, *where*

$$b_t = \max_{0 \le \tilde{c} \le 1} f_t\left(\tilde{c}, (1-\tilde{c}) \max_{\theta \in \Theta_t} \mathrm{E}_t[(b_{t+1} R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right).$$

2. *The* $\arg\max$ $\tilde{c}$ *is the optimal consumption rate* (c/w), *so* $1-\tilde{c}$ *is saving rate out of wealth.*

- Similar to classic optimal portfolio problem of Samuelson (1969), Hakansson (1970), etc.

Proof

- $V_T(w) = b_T w$ by assumption.
- If true for $t+1$, so $V_{t+1}(w) = b_{t+1}w$, then

$$\begin{aligned}
V_t(w) &= \max_{\substack{0\le c\le w\\ \theta\in\Theta_t}} f_t\left(c, \mathrm{E}_t[V_{t+1}(R_{t+1}(\theta)(w-c))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right)\\
&= \max_{\substack{0\le c\le w\\ \theta\in\Theta_t}} f_t\left(c, \mathrm{E}_t[(b_{t+1}R_{t+1}(\theta)(w-c))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right)\\
&= \max_{0\le c\le w} f_t\left(c, (w-c)\max_{\theta\in\Theta_t}\mathrm{E}_t[(b_{t+1}R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right)\\
&= w\max_{0\le \tilde{c}\le 1} f_t\left(\tilde{c}, (1-\tilde{c})\max_{\theta\in\Theta_t}\mathrm{E}_t[(b_{t+1}R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right)\\
&= b_t w,
\end{aligned}$$

so true for t as well.

Definition of EIS for general recursive preferences

- Recall: if no uncertainty, then $\psi = \mathrm{EIS} = \frac{\partial \log(c_{t+1}/c_t)}{\partial \log R_{f,t}}$.
- Assume no uncertainty between t and $t+1$, with risk-free rate $R_{f,t} = R$. Set $b = b_{t+1}$, known at t.
- By Bellman, $\tilde{c}_t = \arg\max_{0 \le c \le 1} f(c, bR(1-c))$.
- By budget constraint, $\frac{c_{t+1}}{c_t} = \frac{w_{t+1}\tilde{c}_{t+1}}{w_t \tilde{c}_t} = \frac{R(1-\tilde{c}_t)\tilde{c}_{t+1}}{\tilde{c}_t}$.
- Since b_{t+1}, $\tilde{c}_{t+1}$ unaffected by (predetermined) $R_{f,t}$, get

$$\psi = \frac{\partial \log(R(1-\tilde{c}_t)\tilde{c}_{t+1}/\tilde{c}_t)}{\partial \log R} = 1 - \frac{R}{(1-\tilde{c}_t)\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial R}.$$

Definition of EIS for general recursive preferences

Definition 3.1

Let $f_t(c, v)$ be a smooth, strictly quasi-concave, homothetic aggregator. Then the *elasticity of intertemporal substitution* of f_t is

$$\psi_t = \frac{\mathrm{d}\log[R(1-\tilde{c})/\tilde{c}]}{\mathrm{d}\log R} = \frac{\tilde{c}}{1-\tilde{c}} \frac{\mathrm{d}[R(1-\tilde{c})/\tilde{c}]}{\mathrm{d}R} = 1 - \frac{R}{(1-\tilde{c})\tilde{c}} \frac{\mathrm{d}\tilde{c}}{\mathrm{d}R},$$

where $\tilde{c}(R) \equiv \arg\max_{0 \le c \le 1} f_t(c, R(1-c))$ **and the derivatives are evaluated at the point** R^* **defined by** $\tilde{c}_t = \tilde{c}(R^*)$.

- Key observations:
  1. EIS is defined by aggregator f alone (but depends on R^*).
  2. Since $0 < \tilde{c} < 1$ and $\rho > 0$, we have $\psi \gtrless 1$ according as $\partial \tilde{c}/\partial R \lessgtr 0$.
  3. R^* is just a number; it does not need to be risk-free rate.

Definition of "bad news"

- Recall value function is $V_t(w) = b_t w$, where

$$b_t = \max_{0\le\tilde{c}\le 1} f_t\left(\tilde{c}, (1-\tilde{c})\max_{\theta\in\Theta_t} \mathrm{E}_t[(b_{t+1}R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}\right).$$

- Continuation value is captured by

$$\rho_t := \max_{\theta\in\Theta_t} \mathrm{E}_t[(b_{t+1}R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}.$$

- Hence define

Bad news at t = Exogenous change that decreases ρ_t.

Main result

Theorem

Consider an investor with EIS smaller than 1. Let $1 - \tilde{c}_s$ be the saving rate out of wealth at time s without the bad news at t, where $\tilde{c}_s$ is consumption rate. Let $1 - \tilde{c}'_s$ be the saving rate with the bad news. Then

$$1 - \tilde{c}'_s \geq 1 - \tilde{c}_s$$

for all $s \leq t$. (The opposite inequality is true when $\mathrm{EIS} > 1$.)

Implication: Can identify whether $\mathrm{EIS} \gtrless 1$ by looking at the response of saving rate to bad news:

- Savings $\uparrow$ with bad news $\implies \mathrm{EIS} < 1$,
- Savings $\downarrow$ with bad news $\implies \mathrm{EIS} > 1$.

Outline of proof

1. Since $\rho_t = \max_{\theta \in \Theta_t} \mathrm{E}_t[(b_{t+1} R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}$, ρ_t is monotonic in b_{t+1}.
2. Since $b_t = \max_{\tilde{c}} f(\tilde{c}, \rho_t(1-\tilde{c}))$, b_t is monotonic in ρ_t.
3. Hence if $\rho_t' < \rho_t$ (bad news at t), then $\rho_s' < \rho_s$ and $b_s' < b_s$ for all $s \le t$.
4. Since

$$\tilde{c}_s = \arg\max_{0 \le \tilde{c} \le 1} f(\tilde{c}, \rho_s(1-\tilde{c})),$$

if $\mathrm{EIS} < 1$, then $\tilde{c}_s \downarrow$, so $1 - \tilde{c}_s \uparrow$. (Recall our definition:)

$$EIS = \frac{\mathrm{d} \log[R(1-\tilde{c})/\tilde{c}]}{\mathrm{d} \log R} = 1 - \frac{R}{(1-\tilde{c})\tilde{c}} \frac{\mathrm{d}\tilde{c}}{\mathrm{d}R},$$

Example of "bad news"

The following are bad news.

1. Increase in risk aversion,
2. Investment opportunity shrinks,
3. Asset returns become riskier,
4. Expected returns decrease,
5. Increase in ambiguity aversion,
6. Investor becomes more uncertain.

Increase in risk aversion

- Recall $\rho_t = \max_{\theta \in \Theta_t} \mathrm{E}_t[(b_{t+1}R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}$.
- For any fixed random variable $X > 0$, can show

$$g(r) := \begin{cases} \mathrm{E}[X^r]^{\frac{1}{r}}, & (r \neq 0) \\ \exp(\mathrm{E}[\log X]) & (r = 0) \end{cases}$$

  is increasing in $r \in \mathbb{R}$.
- Hence ρ_t is decreasing in γ_t, so $\gamma_t \uparrow$ is bad news (lowers continuation utility)

Investment opportunity shrinks

- Recall $\rho_t = \max_{\theta \in \Theta_t} \mathrm{E}_t[(b_{t+1} R_{t+1}(\theta))^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}$.
- Since taking max, the larger the portfolio constraint Θ_t is, the larger ρ_t is.
- Hence shrinking Θ_t is bad news (lowers continuation utility)

Asset returns become riskier/expected returns decline

- Recall

$$\frac{1}{1-\gamma_t}\rho_t^{1-\gamma_t} = \max_{\theta\in\Theta_t} \frac{1}{1-\gamma_t} \mathrm{E}_t[(b_{t+1}R_{t+1}(\theta))^{1-\gamma_t}].$$

- Assume asset returns $\mathbf{R}_{t+1} = (R_{t+1}^1, \dots, R_{t+1}^J)$ become riskier, so $\mathrm{E}\left[\mathbf{R}'_{t+1} \,\middle|\, \mathbf{R}_{t+1}\right] \le \mathbf{R}_{t+1}$ (second order stochastic domination).
- By Jensen's inequality, can show

$$\frac{1}{1-\gamma_t}(\rho'_t)^{1-\gamma_t} \le \frac{1}{1-\gamma_t}\rho_t^{1-\gamma_t},$$

and hence $\rho'_t \le \rho_t$.

- Hence increasing riskiness of asset returns is bad news.

Ambiguity aversion

- So far, considered CRRA certainty equivalent $\mathcal{M}_t(U) = u_t^{-1}(\mathrm{E}_t[u_t(U)])$, where $u_t(x) = \frac{1}{1-\gamma_t}x^{1-\gamma_t}$.
- Consider

$$\mathcal{M}_t(U_{t+1}) = v_t^{-1}(\mathrm{E}_{\mu_t}[v_t(u_t^{-1}(\mathrm{E}_{\pi_t}[u_t(U_{t+1})]))]),$$

where $v_t(x) = \frac{1}{1-\eta_t}x^{1-\eta_t}$ ($\eta_t > 0$: ambiguity aversion), π_t: prior on distribution of asset returns, μ_t: prior on prior (Hayashi & Miao, 2011; Ju & Miao, 2012).

- If $\eta_t = \infty$, then

$$\mathcal{M}_t(U_{t+1}) = u_t^{-1}\left(\min_{\pi_t \in \mathcal{P}_t} \mathrm{E}_{\pi_t}[u_t(U_{t+1})]\right),$$

multi-priors model (Gilboa & Schmeidler, 1989; Epstein & Schneider 2003; Hayashi, 2005).

Increase in ambiguity aversion or more uncertainty

- By the same argument as before, increasing ambiguity aversion η_t is a bad news.
- In the multi-priors case, since

$$\mathcal{M}_t(U_{t+1}) = u_t^{-1}\left(\min_{\pi_t \in \mathcal{P}_t} \mathrm{E}_{\pi_t}[u_t(U_{t+1})]\right),$$

more uncertainty (expanding the set of priors $\mathcal{P}_t$) is a bad news.

Putting the theory to work

- Data requirements: (broad) measures of wealth, savings or consumption. Panel is a plus (can difference out many unobservables).
- Ideal experiment:
  - See two ex-ante similar individuals, only one gets bad news
  - Compare savings behavior before and after the news is revealed
- Sources of identifying variation:
  - Capital income/wealth taxes
  - Increases in consumption/value-added taxes in the future
  - Other ideas? Are there good ways to use uncertainty/risk aversion results?
- One last note: our model probably describes behavior of pension funds and high net worth individuals reasonably well (unless you don't like homotheticity)...

Conclusion

- EIS is a key parameter in any dynamic model, and implications often depend on $\mathrm{EIS} \gtrless 1$.
- Contribution
  1. Define EIS for general recursive preferences.
  2. Provide a new identification of EIS:

$$\text{Savings rate} \uparrow \text{ after bad news} \iff \mathrm{EIS} < 1,$$

  so there is precautionary saving iff $\mathrm{EIS} < 1$.
- Can we use our result to estimate EIS? Hard to observe (total) wealth, hence so is saving rate. Perhaps look at households retired or close to retirement, for whom human capital wealth is negligible?