Bubble Economics[1]

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[1] This talk is intended for advanced undergraduate students interested in mathematical economics. The content is partly based on my review article Hirano and Toda (2024).

Asset price bubbles

- Bubble: asset price (P) $>$ fundamental value (V)
  - $V =$ present value of dividends (D)
- Casual inspection of modern economic history suggests presence of asset price bubbles
  - Japanese real estate bubble in late 1980s
  - U.S. dot-com bubble in late 1990s
  - U.S. housing bubble in mid 2000s
- Kindleberger (2000) documents 38 bubbly episodes in 1618–1998 period

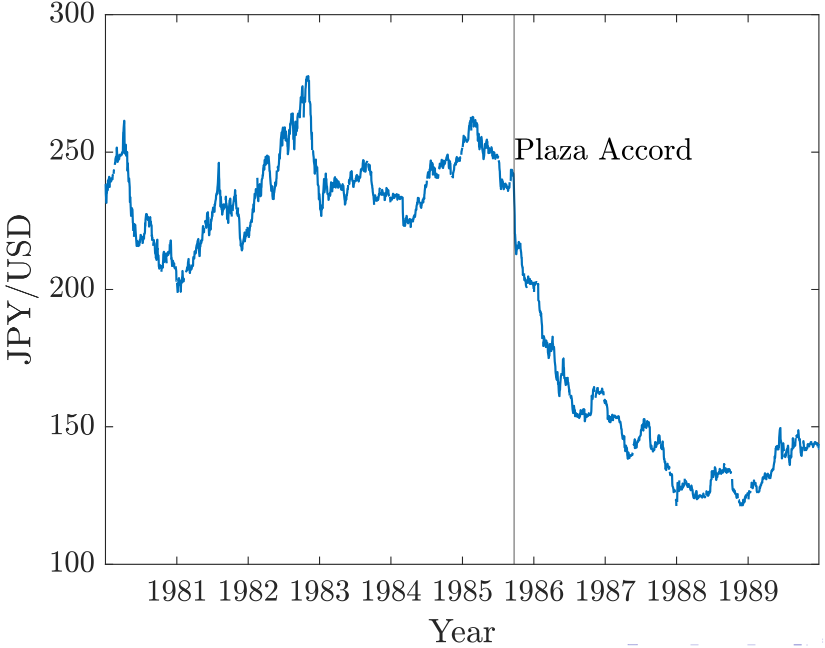
Japan's bubble economy in 1980s

- Japan experienced rapid postwar economic growth
- Coped with oil shocks in 1970s through *shō-ene* (efficiency improvement in energy consumption)
- In 1980s, society was filled with optimism, and Japan was dubbed "number one" (Vogel, 1979)
- A few factors contributed to emergence of real estate bubble
  - 1985 report by National Land Agency titled "Capital Remodeling Plan" predicted Tokyo was destined to become global financial hub (with office space 3,700ha in 1985 $\rightarrow$ 8,700ha in 2000, equivalent to 250 skyscrapers)
  - Interest rate policy by Bank of Japan (BOJ)

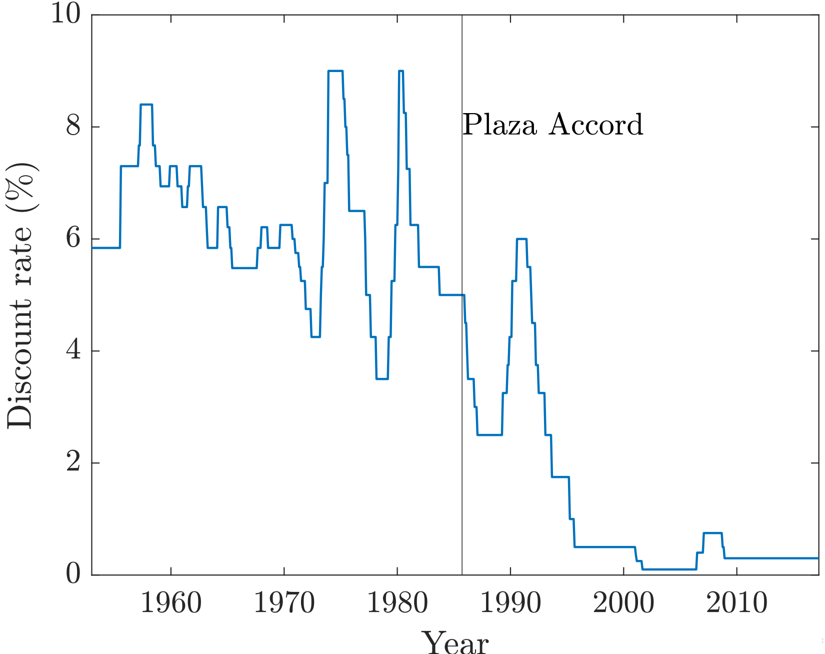
Plaza Accord of September 1985

- In 1980s, U.S. was running trade deficit
- On September 22, 1985, Japan and U.S. agreed to let yen appreciate against dollars (Plaza Accord)
- Exchange rate appreciated from 240 JPY/USD to 150 within a year, causing contraction in manufacturing sector (*endaka fukyō*, or yen-appreciation recession)
- BOJ responded by cutting official discount rate from 5.0% in January 1986 to 2.5% in February 1987 to stimulate economy

JPY/USD exchange rate



BOJ discount rate



Emergence of land bubble

- General optimism and low interest rate environment fueled land speculation
- Land price appreciation accelerated around 1986
- As of 1987, price-rent ratio in Tokyo Marunouchi business district was 20 times of inner city of London (Noguchi, 1990)
- The word "bubble" first appeared in November 26, 1987 issue of *Tōyō Keizai* article titled "Land Price Inflated with Bubbles" by Noguchi
- Easy money also made consumers extravagant: people
  - flocked to expensive restaurants, discos, and ski resorts,
  - drank expensive French wines like Romanée-Conti and Château Latour,
  - bid up 10,000 yen bills along streets to secure taxi rides

Discos during bubble period

"Tower of Bubble"

- Many office buildings were constructed in late 1980s
- Some them were dubbed "Tower of Bubble" due to postmodern architectural style
- Kitaro Watanabe, President of Azabu Building and 6th wealthiest individual in the world in 1990 according to Forbes, borrowed 700 billion yen and possessed 165 office buildings in Tokyo and 6 luxury hotels in Hawaii

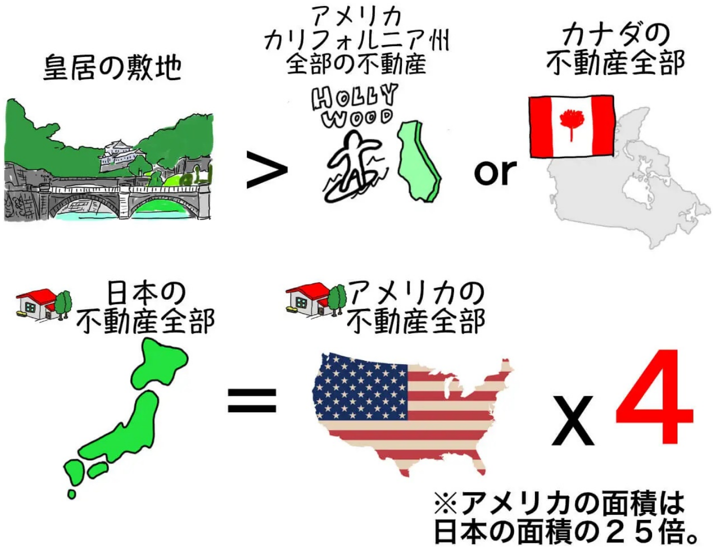
Tokyo Metropolitan Government Building

Joule A

1988 letter to Marusan Security employees

> *The total land value in Japan is estimated to be 4 times of U.S. The land area is 1/25, so the unit price is really 100 times. The land price of the Imperial Palace is about the same as California. Even if the Japanese economy is booming, we cannot expect such an abnormal disparity to be sustainable. Moreover, the Japanese population will decline. Therefore, you should refrain from purchasing housing for the time being.*
>
> *Taro Kaneko, President of Marusan Security*

- Imperial Palace: 1.15 square kilometers; California: 423,970 square kilometers
- Boone (1990) estimated land of Japan to be worth 3 times of U.S.



皇居の敷地
>
アメリカ
カリフォルニア州
全部の不動産
HOLLY
WOOD
or
カナダの
不動産全部
日本の
不動産全部
=
アメリカの
不動産全部
x 4
※アメリカの面積は
日本の面積の２５倍。

Collapse of bubble

- By late 1980s, average households could no longer afford land
- Big social issue known as "land problem"
- BOJ hiked interest rate from 2.5% to 6.0%
- Ministry of Finance introduced Real Estate Loan Total Quantity Restriction
- Easy money dried out and bubble collapsed
  - Essentially, emergence and collapse of bubble were government failures
- Land and stock prices fell by 80% over the following decade
- Many banks failed due to non-performing loans

Definition of bubbles

- Asset dividend $D_t \geq 0$, price $P_t \geq 0$ at $t = 0, 1, \dots$
- With Arrow-Debreu (date-0) price $q_t > 0$, no-arbitrage implies

$$q_t P_t = q_{t+1}(P_{t+1} + D_{t+1}), \quad \text{so}$$

$$P_0 = \sum_{t=1}^{T} q_t D_t + q_T P_T \quad \text{by iteration}$$

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- Letting $T \to \infty$, get

$$P_0 = \underbrace{\sum_{t=1}^{\infty} q_t D_t}_{=:V_0=\text{fundamental value}} + \underbrace{\lim_{T\to\infty} q_T P_T}_{\text{transversality term}}$$

- If $\lim_{T\to\infty} q_T P_T = 0$, transversality condition holds and no bubble; if > 0, bubble

Bubble Characterization Lemma

Lemma

If $P_t > 0$ for all t, asset price exhibits bubble if and only if

$$\sum_{t=1}^{\infty} \frac{D_t}{P_t} < \infty$$

- Hence bubble if and only if sum of dividend yields finite
- Except pure bubble models ($D_t \equiv 0$), bubbles are fundamentally nonstationary phenomena: price must grow faster than dividend

Proof

- By no-arbitrage,

$$q_{t-1}P_{t-1} = q_t(P_t + D_t) \iff \frac{q_{t-1}P_{t-1}}{q_t P_t} = 1 + \frac{D_t}{P_t}$$

- Taking product from $t = 1$ to $t = T$, get

$$\frac{q_0 P_0}{q_T P_T} = \prod_{t=1}^{T} \left(1 + \frac{D_t}{P_t}\right)$$

- Expanding terms and using $1 + x \leq \mathrm{e}^x$, we obtain

$$1 + \sum_{t=1}^{T} \frac{D_t}{P_t} \leq \frac{q_0 P_0}{q_T P_T} \leq \exp\left(\sum_{t=1}^{T} \frac{D_t}{P_t}\right)$$

- Let $T \to \infty$ and use definition of TVC $\square$

Samuelson (1958)'s overlapping generations model

- Time: $t = 0, 1, 2, \ldots$
- At each date t, new generation is born, who lives for two dates (young at t and old at $t + 1$)
- Utility function is

$$U(y_t, z_{t+1}) = (1 - \beta) \log y_t + \beta \log z_{t+1},$$

  where (y_t, z_{t+1}): consumption when young and old
- In addition, at $t = 0$, there is initial old
- So at every t, there is a young agent and an old agent
- Endowments: $a > 0$ when young and $b > 0$ when old

Asset market

- There is a unit supply of intrinsically useless asset (asset that pays no dividends), initially held by the old
- Budget constraints of agent born at time t are

$$\begin{aligned} \text{Young:} \quad & y_t + P_t x_t = a, \\ \text{Old:} \quad & z_{t+1} = b + P_{t+1} x_t, \end{aligned}$$

  where P_t: asset price and x_t: asset holdings
- Note: old liquidate asset before exiting economy

Equilibrium

- Equilibrium is sequence $\{(P_t, x_t, y_t, z_t)\}_{t=0}^{\infty}$ such that
  1. (Agent optimization) generation $t \geq 0$ solves

$$\begin{aligned} \text{maximize} \quad & (1-\beta)\log y_t + \beta \log z_{t+1} \\ \text{subject to} \quad & y_t + P_t x_t = a, \\ & z_{t+1} = b + P_{t+1} x_t \end{aligned}$$

     (For initial old, $z_0 = b + P_0$)
  2. (Commodity market clearing) $y_t + z_t = a + b$
  3. (Asset market clearing) $x_t = 1$
- Note: because old exit economy, in equilibrium, young must demand entire asset
- Two types of equilibria
  - Fundamental equilibrium: $P_t = 0$ for all t (hence $(y_t, z_t) = (a, b)$)
  - Bubbly equilibrium: $P_t > 0$ for all t

Bubbly equilibria

- Let us compute all bubbly equilibria ($P_t > 0$)
- Eliminating x_t from budget constraints, generation t solves

$$\begin{aligned} &\text{maximize} && (1-\beta)\log y_t + \beta \log z_{t+1} \\ &\text{subject to} && y_t + \frac{P_t}{P_{t+1}} z_{t+1} = a + \frac{P_t}{P_{t+1}} b \end{aligned}$$

- Using Cobb-Douglas formula, demand is

$$y_t = (1-\beta)\left(a + \frac{P_t}{P_{t+1}} b\right)$$

- Using budget of young and market clearing $x_t = 1$, get

$$P_t = P_t x_t = a - y_t = a - (1-\beta)\left(a + \frac{P_t}{P_{t+1}} b\right)$$

$$\iff \frac{1}{P_{t+1}} = \frac{\beta a}{(1-\beta)b} \frac{1}{P_t} - \frac{1}{(1-\beta)b}$$

Difference equation for P_t

- Recall difference equation for P_t:

$$\frac{1}{P_{t+1}} = \frac{\beta a}{(1-\beta)b}\frac{1}{P_t} - \frac{1}{(1-\beta)b}$$

- This is linear difference equation in $1/P_t$
- Recall how we solve $x_{t+1} = px_t + q$?
  - If $p = 1$, then $x_t = x_0 + qt$
  - If $p \neq 1$, then $x_t = \frac{q}{1-p} + p^t \left(x_0 - \frac{q}{1-p}\right)$
- If $\beta a = (1-\beta)b$ (case $p = 1$), then

$$\frac{1}{P_t} = \frac{1}{P_0} - \frac{t}{(1-\beta)b} \to -\infty$$

as $t \to \infty$, contradicting $P_t > 0$ for all t

Difference equation for P_t

- If $\beta a \neq (1-\beta)b$ (case $p \neq 1$), then

$$\frac{1}{P_t} = \frac{1}{\beta a - (1-\beta)b} + \left(\frac{\beta a}{(1-\beta)b}\right)^t \left(\frac{1}{P_0} - \frac{1}{\beta a - (1-\beta)b}\right)$$

- If $\beta a < (1-\beta)b$, then

$$\frac{1}{P_t} \to \frac{1}{\beta a - (1-\beta)b} < 0$$

  as $t \to \infty$, contradicting $P_t > 0$ for all t
- If $\beta a > (1-\beta)b$, then easy to show $P_t > 0$ for all t if $0 < P_0 \leq \beta a - (1-\beta)b$, so we have equilibrium
- Hence continuum of bubbly equilibria parameterized by $P_0 \in (0, \beta a - (1-\beta)b]$

Summary of OLG model

- Fundamental equilibrium ($P_t = 0$ always exists)
- If $\beta a \leq (1-\beta)b$, fundamental equilibrium is unique equilibrium
- If $\beta a > (1-\beta)b$, continuum of bubbly equilibria exist, parameterized by initial asset price $P_0 \in (0, \beta a - (1-\beta)b]$
- If $P_0 = \beta a - (1-\beta)b$ (highest possible initial asset price), then $P_t = \beta a - (1-\beta)b$ for all t, so stationary equilibrium

Efficiency of equilibria

- We know from First Welfare Theorem that competitive equilibria are Pareto efficient under some assumptions
- One assumption was locally nonsatiated utility function
- Another implicit assumption was finitely many agents and goods
- But in OLG models, there are infinitely many agents and goods (indexed by $t = 0, 1, 2, \dots$)
- So it is not obvious whether equilibria are efficient

Efficiency of equilibria

- Consider bubbly equilibria
- Because

$$\frac{1}{P_t} = \frac{1}{\beta a - (1-\beta)b} + \left(\frac{\beta a}{(1-\beta)b}\right)^t \left(\frac{1}{P_0} - \frac{1}{\beta a - (1-\beta)b}\right),$$

  each P_t is increasing in P_0
- Consumption of initial old $z_0 = b + P_0$ is increasing in P_0
- Can compute utility of generation t, $U_t := U(y_t, z_{t+1})$, and express using only P_t
- It turns out that U_t is increasing in P_t (hence in P_0), so the bigger the bubble, the more agents are better off

Land as factor of production

- As economies develop and per capita income $\uparrow$, importance of land as factor of production $\downarrow$
- One reason could be humans face biological (quantity) constraints
  - Food intake limited (land produces agricultural products)
  - Leisure time limited (land produces amenities like tennis courts and national parks)

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- One reason could be humans face biological (quantity) constraints
  - Food intake limited (land produces agricultural products)
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- Another could be difference in productivity growth
- Think about quality improvement in
  - "land-intensive products" (e.g., dining, housing, outdoor experience)
  - "high-tech stuff" (e.g., Internet, smart phones, electric vehicles)

GDP share of agriculture decreases with income

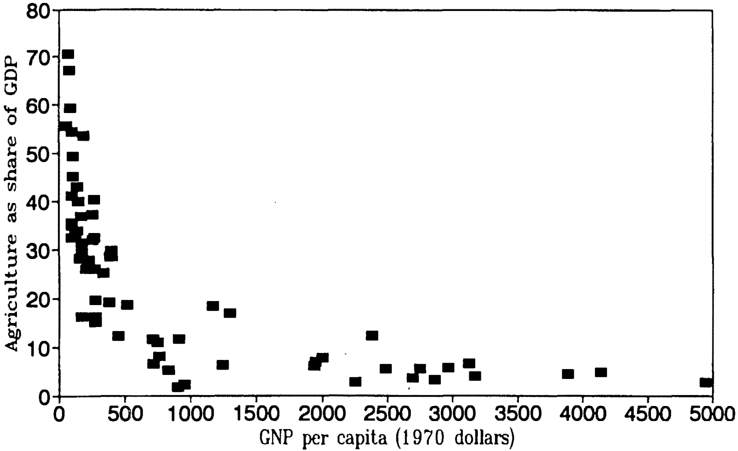


Figure: Echevarria (1997, Figure 2)

Employment share of agriculture decreases over time

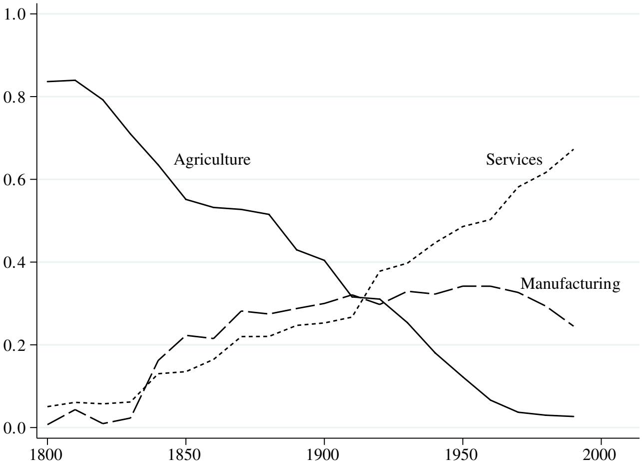


FIGURE 20.1 The share of U.S. employment in agriculture, manufacturing, and services, 1800–2000.

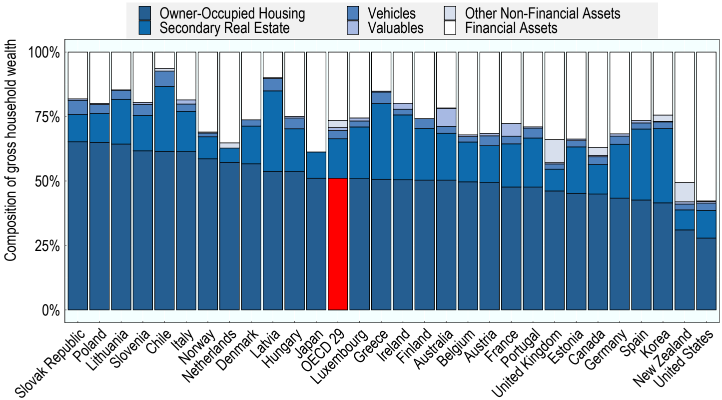
Figure: Acemoglu (2009, Figure 20-1)

Land as store of value

- Land continues to play significant role as store of value
- In many countries, housing wealth is substantial

Figure 2.1. Average decomposition of household assets, 29 OECD countries

2019 or latest available year



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3. Low depreciation (except pollution, erosion, sea level rise)
   - Compare to vehicles, household appliances
4. Non-reproducible
   - Compare to fiat money
5. Property rights well defined
   - Compare to gold, silver

Overlapping generations model

- Generations (agents) are indexed by $t = 0, 1, \dots$
- Agents born at t live for two periods and has utility $U(y_t, z_{t+1})$
  - y_t: consumption when young
  - z_{t+1}: consumption when old
- In addition, at $t = 0$ there are initial old
- Other than infinitely many agents and sequential structure of time, overlapping generations (OLG) model is just a special general equilibrium model

Two-sector growth economy with land

- Two-period OLG model (young & old, constant population)
- Cobb-Douglas utility $(1-\beta)\log y_t + \beta \log z_{t+1}$
- Young have labor 1, old 0
- Initial old own land (unit supply, durable, non-reproducible)
- Two sectors with neoclassical production functions

$$F_{1t}(H,X) = A_{1t}H,$$
$$F_{2t}(H,X) = A_{2t}H^{\alpha}X^{1-\alpha},$$

  where H: labor/human capital, X: land
  - Sector 1: labor-intensive (service, finance, information, etc.)
  - Sector 2: land-intensive (agriculture, construction, etc.)
- Productivity $\{(A_{1t}, A_{2t})\}_{t=0}^{\infty}$ exogenous and deterministic

Equilibrium

- Equilibrium is sequence

$$\{(P_t, r_t, w_t, x_t, y_t, z_t, H_{1t}, H_{2t})\}_{t=0}^{\infty},$$

where P_t: land price, r_t: land rent, w_t: wage, x_t: land holdings, (y_t, z_t): young and old consumption, (H_{1t}, H_{2t}): labor input
- Utility/profit maximization
- Market clearing
  - good
  - land
  - labor

Profit maximization

- Firm j maximizes profit

$$F_{jt}(H, X) - w_t H - r_t X$$

- Assume both sectors active (easy to provide sufficient condition)
- Using $X = 1$, profit maximization is

$$\alpha A_{2t} H_{2t}^{\alpha-1} = w_t = A_{1t} \iff H_{2t} = \alpha^{\frac{1}{1-\alpha}} (A_{2t}/A_{1t})^{\frac{1}{1-\alpha}}$$

- Wage and rent:

$$\begin{aligned} w_t &= A_{1t}, \\ r_t &= (1-\alpha) A_{2t} H_{2t}^{\alpha} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} (A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}} \end{aligned}$$

Utility maximization

- Young maximize utility subject to budget constraints

$$\begin{aligned} \text{Young:} \quad & y_t + P_t x_t = w_t, \\ \text{Old:} \quad & z_{t+1} = (P_{t+1} + r_{t+1}) x_t \end{aligned}$$

- Combine sequential budget constraints to

$$y_t + \frac{1}{R_t} z_{t+1} = w_t,$$

  where $R_t := (P_{t+1} + r_{t+1})/P_t$ is gross return on land
- Because utility Cobb-Douglas, demand is $y_t = (1 - \beta) w_t$

Equilibrium land price

- Because old exit economy, land market clearing implies $x_t = 1$
- Hence equilibrium land price driven by income:

$$P_t = P_t x_t = w_t - y_t = \beta w_t = \beta A_{1t}$$

- Hence rent yield (rent-price ratio) is

$$\frac{r_t}{P_t} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}}}{\beta A_{1t}} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\beta}(A_{2t}/A_{1t})^{\frac{1}{1-\alpha}}$$

- Suppose labor productivity grows faster than land productivity (unbalanced growth, e.g., $A_{1t}/A_{2t} \sim G^t$ with $G > 1$)
- Then $\{r_t/P_t\}$ summable, and land bubble necessarily emerges by Bubble Characterization Lemma ▸ ?

Intuition

- Suppose for simplicity that $A_{1t} = G^t$, $A_{2t} = 1$
- Then rent $r_t = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}} \sim G^{-\frac{\alpha t}{1-\alpha}}$
- Land price $P_t = \beta A_{1t} \sim G^t$
- Hence interest rate

$$R_t = \frac{P_{t+1} + r_{t+1}}{P_t} \sim G > 1$$

- Hence fundamental value of land finite, while land price grows exponentially driven by demand for savings, generating land bubble
- Fore more details see Hirano and Toda (2023)

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