Technological Innovation and Bursting Bubbles[1]

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Bubble and technological innovation

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- Examples:
  - 1720s French Mississippi bubble and British South Sea bubble: Atlantic trade, insurance
  - 1840s British railway mania: steam engine, railway network
  - 1890s British bicycle mania: pneumatic tire
  - 1920s U.S. stock price boom: electricity, consumer durables, automobile, etc.
  - 1990s U.S. dot-com bubble: Internet
  - Now: AI?

Rational asset price bubbles

- Bubble: asset price $(Q) >$ fundamental value (V)
  - $V =$ present value of dividends (D)
- Fundamental difficulty in generating asset price bubbles in real assets
  - Santos and Woodford (1997): bubble impossible if dividends nonnegligible relative to endowments
  - See Hirano and Toda (2024, JME) for illustration
- Theory of rational asset price bubbles attached to dividend-paying assets (including housing) largely underdeveloped
  - See Wilson (1981, JET), Hirano and Toda (2025a, JPE), Hirano and Toda (2025b, PNAS)

This paper

- Macro-finance model of innovation and stock bubble
- Features:
  - Skilled agents choose to work in knowledge-intensive sector or establish new firms
  - Monopolistic competition: firm stocks pay dividends
  - Strength of knowledge spillover determines dividend growth rate
  - Agents expect spillover to eventually weaken (regime switching with absorbing state)

Main results

1. Agents rationally expect boom to eventually end, but bubble ($Q > V$) emerges as unique equilibrium outcome
   - Bubble necessity (Hirano and Toda, 2025a)
2. Long- and short-run effects of stock bubbles
   - Positive feedback between innovation and stock price
   - Despite inevitable collapse, bubble permanently increases output (because technology prevails)
   - Effect on wage inequality temporary
3. Implications for macro-financial modeling
   - Balanced growth is knife-edge (Uzawa, 1961; Schlicht, 2006)
   - Unbalanced growth and bubbles

Related literature

- Rational bubble: Samuelson (1958), Bewley (1980), Tirole (1985), Scheinkman and Weiss (1986), Kocherlakota (1992), Santos and Woodford (1997)
- Rational bubble attached to real assets: Hirano and Toda (2024, 2025a,b)
- Stochastic bubble: Blanchard (1979), Weil (1987)
- Technological innovation and asset boom: Olivier (2000), Pástor and Veronesi (2009)

Model: agents and preferences

- Time: $t = 0, 1, \dots$
- Two period overlapping generations (OLG) model
- Aggregate uncertainty
- Epstein-Zin utility with unit EIS

$$U(c_t^y, c_{t+1}^o) = (1-\beta)\log c_t^y + \beta \log \mathrm{E}_t[(c_{t+1}^o)^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

Model: endowments and dividends

- Young endowed with $e_t > 0$ units of good, old none
- Initial old endowed with unit supply of long-lived asset that pays dividend $D_t > 0$
- $\{(e_t, D_t)\}_{t=0}^{\infty}$ follows some stochastic process
- Budget constraints

$$\begin{aligned} \text{Young:} \quad & c_t^y + Q_t n_t = e_t, \\ \text{Old:} \quad & c_{t+1}^o = (Q_{t+1} + D_{t+1}) n_t, \end{aligned}$$

where Q_t: asset price, n_t: asset holdings

Equilibrium

Definition

Stochastic process $\left\{(Q_t, c_t^y, c_t^o, n_t)\right\}_{t=0}^{\infty}$ is *rational expectations equilibrium* if

1. (Utility maximization) initial old consume $c_0^o = Q_0 + D_0$; for each $t \geq 0$, (c_t^y, n_t, c_{t+1}^o) maximizes utility subject to budget constraints,
2. (Commodity market clearing) for each t, we have $c_t^y + c_t^o = e_t + D_t$,
3. (Asset market clearing) for each t, we have $n_t = 1$.

Unique equilibrium

- Due to unit EIS, optimal consumption of young is

$$c_t^y = (1-\beta)e_t$$

- Young budget constraint and $n_t = 1$ forces

$$Q_t = Q_t n_t = e_t - c_t^y = \beta e_t$$

Proposition

There exists unique rational expectations equilibrium. Asset price is $Q_t = \beta e_t$ and consumption is $(c_t^y, c_t^o) = ((1-\beta)e_t, \beta e_t + D_t)$.

Stochastic discount factor

- Let
$$m_{t\to t+1} = \frac{\partial U/\partial c^o_{t+1}}{\partial U/\partial c^y_t} = \frac{\beta}{1-\beta}\frac{c^y_t (c^o_{t+1})^{-\gamma}}{\mathrm{E}_t[(c^o_{t+1})^{1-\gamma}]}$$
be stochastic discount factor (SDF) between time t and $t+1$
- Using equilibrium conditions,
$$\begin{aligned} m_{t\to t+1} &= \frac{\beta}{1-\beta}\frac{(1-\beta)e_t(\beta e_{t+1}+D_{t+1})^{-\gamma}}{\mathrm{E}_t[(\beta e_{t+1}+D_{t+1})^{1-\gamma}]} \\ &= \frac{Q_t(Q_{t+1}+D_{t+1})^{-\gamma}}{\mathrm{E}_t[(Q_{t+1}+D_{t+1})^{1-\gamma}]} \end{aligned}$$
- Useful later

No-arbitrage condition

- Let $m_{t\to t+1}$ be SDF between t and $t+1$
- Let $m_{t\to t+s} = m_{t\to t+1} \times \cdots \times m_{t+s-1\to t+s}$ be SDF between t and $t+s$
- No-arbitrage condition is

$$Q_t = \mathrm{E}_t[m_{t\to t+1}(Q_{t+1} + D_{t+1})]$$

- Iteration yields

$$Q_0 = \mathrm{E}_0 \sum_{s=1}^{t} m_{0\to s} D_s + \mathrm{E}_0[m_{0\to t} Q_t]$$

Fundamental value and bubble

- Letting $t \to \infty$, get

$$Q_0 = \underbrace{\mathrm{E}_0 \sum_{s=1}^{\infty} m_{0\to s} D_s}_{=:V_0} + \underbrace{\lim_{t\to\infty} \mathrm{E}_0[m_{0\to t} Q_t]}_{=:B_0},$$

where

  - V_0: fundamental value,
  - B_0: bubble

- By definition, no bubble if and only if

$$\lim_{t\to\infty} \mathrm{E}_0[m_{0\to t} Q_t] = 0$$

Emergence of stochastic bubbles

- We now put more structure to derive stochastic bubbles

Assumption

There are two states denoted by u, b. Letting $z_t \in \{u, b\}$ denote state at time t, transition probabilities given by

$$\Pr[z_{t+1} = u \mid z_t = u] = \pi \in (0, 1),$$
$$\Pr[z_{t+1} = b \mid z_t = b] = 1.$$

- State u persists with probability π
- State b absorbing

State b exhibits balanced growth

Assumption

For any τ, conditional on $z_\tau = b$, sequence $\{(e_t, D_t)\}_{t=\tau}^{\infty}$ is deterministic and $e_{t+1}/e_t = D_{t+1}/D_t$ for all $t \geq \tau$.

State b exhibits balanced growth

Assumption

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Proposition

Once state b is reached, no bubble: $Q_t = V_t$.

State b exhibits balanced growth

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For any τ, conditional on $z_\tau = b$, sequence $\{(e_t, D_t)\}_{t=\tau}^{\infty}$ is deterministic and $e_{t+1}/e_t = D_{t+1}/D_t$ for all $t \geq \tau$.

Proposition

Once state b is reached, no bubble: $Q_t = V_t$.

- Intuition: $Q_t = \beta e_t$ grows with endowment
- In state b, uncertainty resolved and gross risk-free rate

$$R_{t+1} = \frac{\beta e_{t+1} + D_{t+1}}{\beta e_t} = \frac{e_{t+1}}{e_t}\left(1 + \frac{1}{\beta}\underbrace{\frac{D_{t+1}}{e_{t+1}}}_{\text{constant}}\right)$$

  exceeds endowment growth, so discounting rules out bubbles

Condition for bubbles in state u

Assumption

Conditional on time $t-1$ information, endowment e_t and dividend D_t depend only on state $z_t \in \{u, b\}$.

Theorem

For $z \in \{u, b\}$, let (e_t^z, D_t^z) be value of (e_t, D_t) conditional on $z_0 = \cdots = z_{t-1} = u$ and $z_t = z$ and let $c_t^z := \beta e_t^z + D_t^z$. If $z_0 = u$, then there is a bubble at $t = 0$ if and only if

$$\sum_{t=1}^{\infty} D_t^u / e_t^u < \infty,$$

$$\sum_{t=1}^{\infty} (c_t^b / c_t^u)^{1-\gamma} < \infty.$$

Intuition and implications

1. Noting $Q_t = \beta e_t$, $\sum D_t^u / e_t^u < \infty$ implies $Q_t^u / D_t^u \to \infty$. Hence bubble can be understood as temporary deviation from balanced growth and explosive dynamics in P/D ratio
2. Equilibrium is unique. Hence (under these conditions) asset price bubble is necessity, not possibility
3. Conditions for stochastic bubbles stronger than deterministic case (Montrucchio, 2004, Proposition 7); if $\gamma < 1$, need crash to be larger the longer the bubble lasts

Model with innovation and intangible capital

- Endowment economy highly stylized
- We extend toy model to production, innovation, intangible capital (Grossman and Helpman, 1991)
  - R&D
  - Monopolistic competition

Agents and preferences

- Basically same as toy model
  - Two-period OLG model
  - Epstein-Zin preferences
- Mass $L > 0$ unskilled agents work in consumption good sector
- Mass $H > 0$ skilled agents either
  - Work in knowledge-intensive intermediate good firms, or
  - Engage in R&D and establish new firms

Consumption good sector

- Representative firm produces output (consumption good)

$$Y_t = F(A_{Xt}X_t, A_{Lt}L_t),$$

where

  - F: neoclassical production function (e.g., CES)
  - X_t: knowledge-intensive good, L_t: unskilled labor
  - A_{Xt}, A_{Lt}: factor-augmenting productivities

- Maximizes profit

$$Y_t - P_tX_t - w_{Lt}L_t,$$

where

  - P_t: price of knowledge-intensive good
  - w_{Lt} unskilled wage

- Zero profit

Knowledge-intensive good sector

- Representative firm produces knowledge-intensive good

$$X_t = n_t^{1-1/\theta} \left(\int_0^{n_t} [x_t(j)]^\theta \, \mathrm{d}j \right)^{1/\theta},$$

where
  - n_t: "knowledge"
  - $x_t(j)$: knowledge-intensive intermediate good produced by firm j
  - $\theta \in (0, 1)$: elasticity parameter
- Maximizes profit

$$P_t X_t - \int_0^{n_t} p_t(j) x_t(j) \, \mathrm{d}j$$

- Zero profit

Knowledge-intensive intermediate good sector

- Intermediate goods differentiated by $j \in [0, n_t]$
- Skilled labor produces intermediate good $1 : 1$
- Firm j maximizes profit

$$d_t(j) = (p_t(j) - w_{Ht})x_t(j)$$

by setting $p_t(j)$ (monopolistic competition), taking wage w_{Ht} and demand $x_t(j)$ as given

- Profit $d_t(j)$ paid as dividend to firm j stock

R&D sector

- New intermediate good varieties created through R&D
- 1 unit of skilled labor $\rightarrow$ an_t new varieties (firms)
- Founder sells stocks (claim to monoply profits) at IPO
- Hence indifference condition

$$w_{Ht} = Q_t a n_t,$$

where $Q_t = q_t(j)$ stock price

Equilibrium

- First-order conditions of consumption good

$$P_t = F_X(A_{Xt}X_t, A_{Lt}L)A_{Xt},$$
$$w_{Lt} = F_L(A_{Xt}X_t, A_{Lt}L)A_{Lt}$$

- FOC of intermediate good j

$$p_t(j) = P_t(X_t/n_t)^{1-\theta}x_t(j)^{\theta-1}$$

- Hence demand for intermediate good j

$$x_t(j) = (X_t/n_t)(p_t(j)/P_t)^{-\frac{1}{1-\theta}}$$

Equilibrium

- Monopolistic competition implies

$$p_t(j) = w_{Ht}/\theta$$

- Because $p_t(j)$ common across j, so is $x_t(j) = x_t$
- Dividend

$$d_t(j) = (p_t(j) - w_{Ht})x_t(j) = \frac{1-\theta}{\theta} w_{Ht} x_t$$

  also common across j
- Hence may focus on symmetric equilibrium $Q_t = q_t(j)$

Optimal consumption and saving

- Skilled agents indifferent between working and R&D:

$$a n_t Q_t = w_{Ht}$$

- Budget constraints of type $i \in \{H, L\}$

$$\begin{aligned} &\text{Young:} && c_{it}^{y} + Q_t n_{it} = w_{it}, \\ &\text{Old:} && c_{i,t+1}^{o} = (Q_{t+1} + D_{t+1}) n_{it}. \end{aligned}$$

- Optimal consumption $c_{it}^{y} = (1 - \beta) w_{it}$, so stock demand

$$\begin{aligned} n_{Ht} &= \beta w_{Ht} / Q_t = \beta a n_t, \\ n_{Lt} &= \beta w_{Lt} / Q_t = (w_{Lt} / w_{Ht}) \beta a n_t. \end{aligned}$$

Market clearing

- Let $\phi_t \in (0, H]$ be fraction of skilled agents working
- Market clearing for skilled labor:

$$X_t = n_t x_t = \phi_t H$$

- Fraction $1 - \phi_t$ engage in R&D, so

$$n_{t+1} = (1 + a(1 - \phi_t)H)n_t$$

- Stock market clearing:

$$\underbrace{n_{t+1}}_{\text{supply}} = \underbrace{Hn_{Ht} + Ln_{Lt}}_{\text{demand}}$$

Equilibrium

Proposition

There exists unique equilibrium. ϕ_t *solves*

$$\frac{1}{aH} = \phi_t - 1 + \beta + \beta \left[\theta \frac{A_{Xt} H}{A_{Lt} L} g \left(\frac{A_{Xt} H}{A_{Lt} L} \phi_t \right) \right]^{-1}.$$

Equilibrium prices are

Knowledge-intensive good price:	$P_t = p_t(j) = F_X A_{Xt},$
Skilled wage:	$w_{Ht} = \theta F_X A_{Xt},$
Unskilled wage:	$w_{Lt} = F_L A_{Lt},$
Stock price:	$Q_t = \frac{w_{Ht}}{an_t} = \frac{\theta}{an_t} F_X A_{Xt},$

where F_X, F_L *are evaluated at* $(A_{Xt} H \phi_t, A_{Lt} L)$.

Production function and productivities

- Specialize production function and productivities (A_{Xt}, A_{Lt})
- Production function is CES:

$$F(X, L) = \left(\alpha X^{1-\rho} + (1-\alpha) L^{1-\rho}\right)^{\frac{1}{1-\rho}},$$

  where $\alpha \in (0, 1)$ and $1/\rho$ is elasticity of substitution
- As before, two states $z \in \{u, b\}$

Knowledge spillover

Assumption

There exist constants $A_X, A_L > 0$ *and* $\xi_u, \xi_b, \lambda_u, \lambda_b \geq 0$ *such that*

$$(A_{Xt}, A_{Lt}) = (A_X n_t^{\xi_{z_t}}, A_L n_t^{\lambda_{z_t}}).$$

Furthermore,

$$\psi := (\xi_u - \lambda_u)(\rho - 1) > 0,$$
$$\lambda_u > \lambda_b = \xi_b.$$

- State b is balanced growth ($\xi_b = \lambda_b$)
- Suffices to assume $1/\rho < 1$ (complement) and $\xi_u > \lambda_u > \lambda_b$ (spillover stronger in state u and in knowledge-intensive good sector)

Equilibrium

- Equilibrium conditions

$$u: \quad \frac{1}{aH} = \phi_t - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha}\left(\frac{A_X H}{A_L L}\right)^{\rho-1} n_t^{(\xi_{z_t}-\lambda_{z_t})(\rho-1)} \phi_t^{\rho},$$

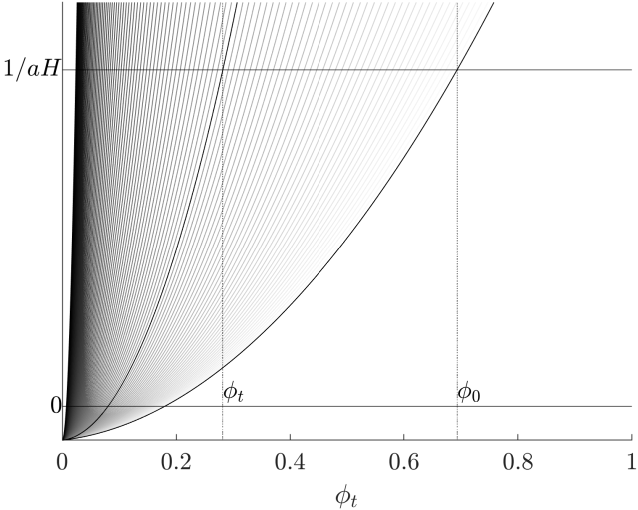
$$b: \quad \frac{1}{aH} = \phi_b - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha}\left(\frac{A_X H}{A_L L}\right)^{\rho-1} \phi_b^{\rho}.$$

Proposition

Under maintained assumptions, following statements are true.

1. *Conditional on staying in state u, $\{\phi_t\}$ monotonically converges to zero and knowledge n_t asymptotically grows at rate $G_u := 1 + aH$.*
2. *In state b, $\{\phi_t\}$ is constant at ϕ_b and knowledge n_t grows at rate $G_b := 1 + a(1-\phi_b)H < G_u$.*

Dynamics of ϕ_t



Inevitable emergence of stock price bubbles

- Equilibrium dynamics reduces to toy model

Theorem

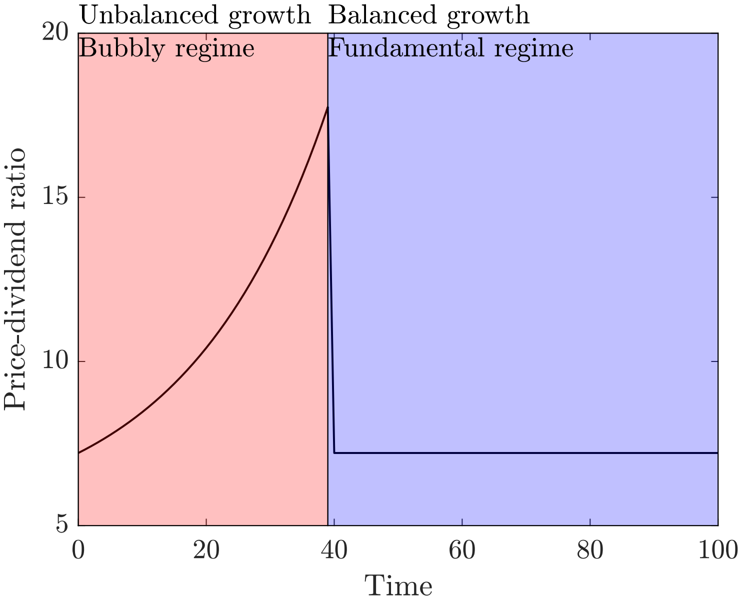
Suppose production function CES and relative risk aversion is $\gamma < 1$. Let Q_t be stock price in unique equilibrium and V_t fundamental value. Then

1. *In state $z_t = u$, stock price exhibits a bubble: $Q_t > V_t$ and price-dividend ratio Q_t/D_t grows exponentially.*
2. *In state $z_t = b$, stock price reflects fundamentals: $Q_t = V_t$ and price-dividend ratio Q_t/D_t is constant.*

Intuition and implications

1. Temporary unbalanced technological growth driven by regime switching and some conditions on elasticities necessarily generate stock price bubble
2. Dynamics of price-dividend ratio markedly different:
   - In state u, exponential growth
   - In state b, constant
3. Stock price bubble can be understood as temporary deviation from balanced growth; agents willing to buy overpriced stocks despite expecting collapse

Numerical example



Implication for skilled wage

- What is effect of stock price bubble?
- Stock price Q_t pushed above fundamental value V_t
- Indifference condition $w_{Ht} = an_t Q_t$, so $Q_t \uparrow \implies w_{Ht} \uparrow$

Observation

The stock market bubble tends to increase the skilled wage.

Implication for innovation

- Skilled wage $w_{Ht} = \theta F_X A_{Xt}$, where F evaluated at $(A_{Xt} H \phi_t, A_{Lt} L)$
- Noting F concave and (A_{Xt}, A_{Lt}) predetermined, $w_{Ht} \uparrow \implies \phi_t \downarrow$
- Hence fraction of skilled agents in R&D, $1 - \phi_t \uparrow$

Observation

The stock market bubble tends to promote innovation.

Implication for wage inequality

- Unskilled wage $w_{Lt} = F_L A_{Lt}$, where F evaluated at $(A_{Xt} H \phi_t, A_{Lt} L)$
- Hence $\phi_t \downarrow \implies w_{Lt} \downarrow$ (fixing n_t)

Observation

The stock market bubble tends to increase the wage gap between skilled and unskilled agents.

Implication for short-run output

- Assume state switches from u to b (bubble bursts) at t
- After burst, output is

$$Y_t = F(A_X H \phi_b, A_L L) n_t^{\lambda_b} \sim n_t^{\lambda_b}$$

- Before burst, output is

$$Y_{t-1} = F(A_{X,t-1} H \phi_t, A_{L,t-1} L) \sim n_{t-1}^{\lambda_u}$$

- Hence output growth has order of magnitude $n_t^{\lambda_b - \lambda_u}$

Observation

The longer the stock market bubble lasts (with higher n_t), the more severe the economic contraction when it bursts.

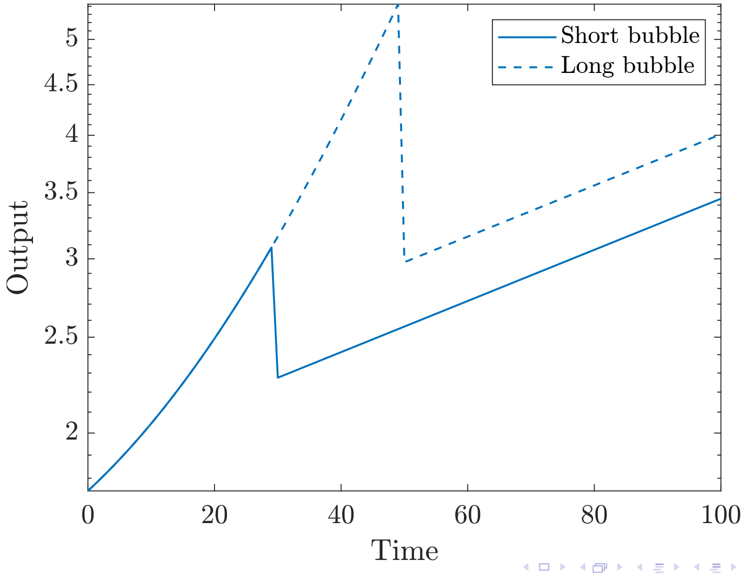
Implication for long-run output

- In long run, state is b and output $Y_t \sim n_t^{\lambda_b}$
- n_t larger (more innovation) the longer bubble lasts

Observation

The stock market bubble tends to increase the output in the long run.

Dynamics of output



Implication for long-run wages

- In long run, $\phi_t = \phi_b$ constant
- Hence relative wage

$$\frac{w_{Ht}}{w_{Lt}} = \theta \frac{F_X}{F_L} \frac{A_X}{A_L}$$

constant

Observation

The stock market bubble tends to increase wages in the long run but does not affect the wage gap between skilled and unskilled agents.

Balanced growth is knife-edge

- In macro, there is strong presupposition in balanced growth
- But balanced growth is knife-edge (Uzawa (1961) steady state growth theorem)

Proposition

Assume only Epstein-Zin utility and neoclassical production function F. Then price-dividend ratio Q_t/D_t is constant over time if and only if either relative productivity A_{Xt}/A_{Lt} is constant or production function F is Cobb-Douglas.
In particular, in our setting, parameters need to satisfy

$$\psi = (\xi_u - \lambda_u)(\rho - 1) = 0.$$

Conclusion

- Any balanced growth model is knife-edge theory
- Once we adopt unbalanced growth (here due to uneven technological spillover), asset price bubble becomes necessity
- Tight connection between technological innovation and stock bubble
- Innovation-driven stock bubble has many benefits (e.g., higher long-run output because more innovation) despite inevitable collapse

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