Susceptible-Infected-Recovered (SIR) Dynamics of COVID-19 and Economic Impact

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Up-to-date analysis:

https://sites.google.com/site/aatoda111/misc/covid19

Coronavirus Disease 2019

- Respiratory syndrome caused by new stain of coronavirus was reported in Wuhan, China in December 2019
- Named Coronavirus Disease 2019 (COVID-19)
  - Symptoms include fever, dry cough, tiredness, and joint pain, which resemble seasonal influenza
  - Vast majority of cases ($> 99\%$?) are mild and recover with no special treatment ▸ Diamond Princess
  - Severe cases ($< 1\%$?) lead to acute respiratory distress syndrome (ARDS) and require intensive care (intubation, ventilation, fluid management, etc.)

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  - Severe cases ($< 1\%$?) lead to acute respiratory distress syndrome (ARDS) and require intensive care (intubation, ventilation, fluid management, etc.)
- Early containment appears to be successful (so far) in China but have failed in other countries
- As of March 31, 2020, there are 780,000 cases reported and 37,000 deaths globally; different from 2003 SARS SARS

What should we do?

- Individuals need rational expectations about the course of epidemic to make everyday decisions
  - Should I social-distance, and for how long?
  - Should I buy toilet paper and pasta?
  - Should I buy or sell stocks?
- Policy makers need information to design policy
  - Should we close schools, bars, restaurants, and other businesses?
  - Should we restrict travel?
  - When and how long?
- All of above require up-to-date data and out-of-sample predictions

This paper

1. Using daily cross-country data on COVID-19 cases, estimates parameters of mathematical epidemic model
2. Makes out-of-sample predictions: in US,
   - Peak comes in 31 days
   - At peak, 38% of population infected simultaneously

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1. Using daily cross-country data on COVID-19 cases, estimates parameters of mathematical epidemic model
2. Makes out-of-sample predictions: in US,
   - Peak comes in 31 days
   - At peak, 38% of population infected simultaneously
3. Discusses optimal mitigation policy: based on model,
   - Current drastic social distancing measures suboptimal: can delay peak but no effect on *size* of peak (unless permanent)
   - Optimal policy is to social-distance only when sufficiently many people ($\sim 1\%$) are infected; under optimal policy, peak reduces to 5.6%
   - Intuition: need to acquire herd immunity to end epidemic; premature drastic measures prevent this
4. Application to economic impact (stock market)

Susceptible-Infected-Recovered (SIR) model

- SIR model (Kermack & McKendrick, 1927) is simplest epidemic model
- Society consists of N individuals, among whom
  - S are *susceptible* (not immune)
  - I are *infected* (can infect non-immune individuals)
  - R are *recovered* (immune or dead)
- Model can be extended to allow for features such as
  - Exposed individuals (those already infected, but not able to infect others yet)
  - Multiple regions with inflow/outflow

Susceptible-Infected-Recovered (SIR) model

- By accounting, $N = S + I + R$. Define
  - $x = S/N$: fraction of susceptible
  - $y = I/N$: fraction of infected
  - $z = R/N$: fraction of recovered
- Key differential equations:

$$\begin{aligned} \dot{x} &= -\beta xy, \\ \dot{y} &= \beta xy - \gamma y, \\ \dot{z} &= \gamma y. \end{aligned}$$

- Interpretations:
  - Infected (y) meet susceptible (x) and transmit disease at certain *transmission rate* β
  - Infected (y) recover (or die) at certain *recovery rate* γ

Some qualitative observations of SIR model

- Suppose $x_0, y_0 > 0$. Then

$$\dot{y} = (\beta x - \gamma)y \begin{cases} \leq 0 & \text{if } \beta x \leq \gamma, \\ > 0 & \text{if } \beta x > \gamma. \end{cases}$$

- Hence infection rate y single-peaked and
  - no epidemic if $\beta x_0 \leq \gamma$
  - epidemic if $\beta x_0 > \gamma$
- **Definition:** society has herd immunity if no-epidemic condition $\beta x \leq \gamma$ holds
- Reproduction number $R = \beta/\gamma$ key parameter

Analytical solution to SIR model

- Despite nonlinearities (x-y interaction), SIR model has parametric closed-form solution
- Convenient for estimation & prediction

Proposition (Harko et al.(2014))

Let $x(0) = x_0 > 0$, $y(0) = y_0 > 0$, $z(0) = z_0 \geq 0$ *be given, where* $x_0 + y_0 + z_0 = 1$. *Then SIR model has parametric solution*

$$\begin{aligned} x(t) &= x_0 v, \\ y(t) &= \frac{\gamma}{\beta} \log v - x_0 v + x_0 + y_0, \\ z(t) &= -\frac{\gamma}{\beta} \log v + z_0, \end{aligned}$$

where

$$t = \int_v^1 \frac{\mathrm{d}\xi}{\xi(\beta x_0(1-\xi) + \beta y_0 + \gamma \log \xi)}.$$

Theoretical properties

Proposition

The followings hold in SIR model:

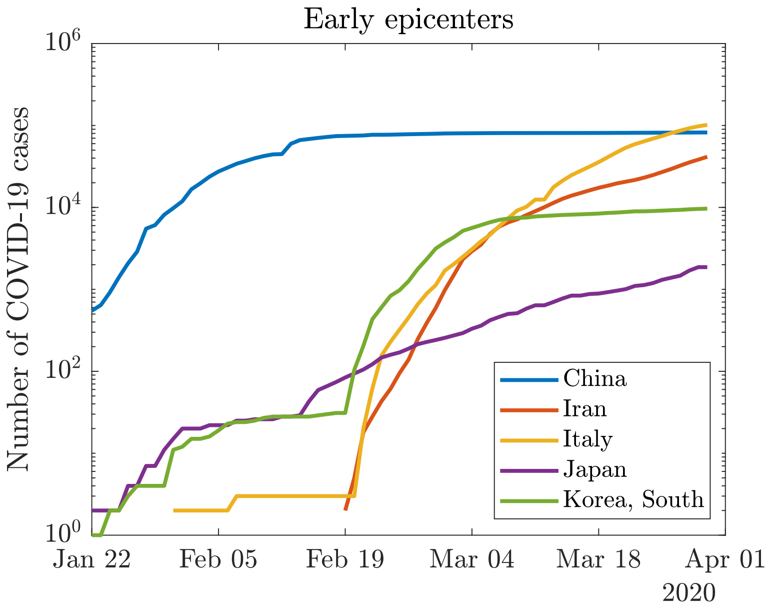
1. *In the long run, fraction $v^* \in (0,1)$ of susceptible individuals infected (fraction $1 - v^*$ infected), where v^* is unique solution to*
$$x_0(1 - v) + y_0 + \frac{\gamma}{\beta} \log v = 0.$$
2. *If $\beta x_0 > \gamma$, peak of epidemic happens when $\beta x(t_{\max}) = \gamma$, at which point fraction*
$$y_{\max} = y(t_{\max}) = \frac{\gamma}{\beta} \log \frac{\gamma}{\beta x_0} - \frac{\gamma}{\beta} + x_0 + y_0$$
of population is infected. The maximum infection rate $y_{\max}$ is increasing in x_0, y_0, and reproduction number $R = \beta/\gamma$.

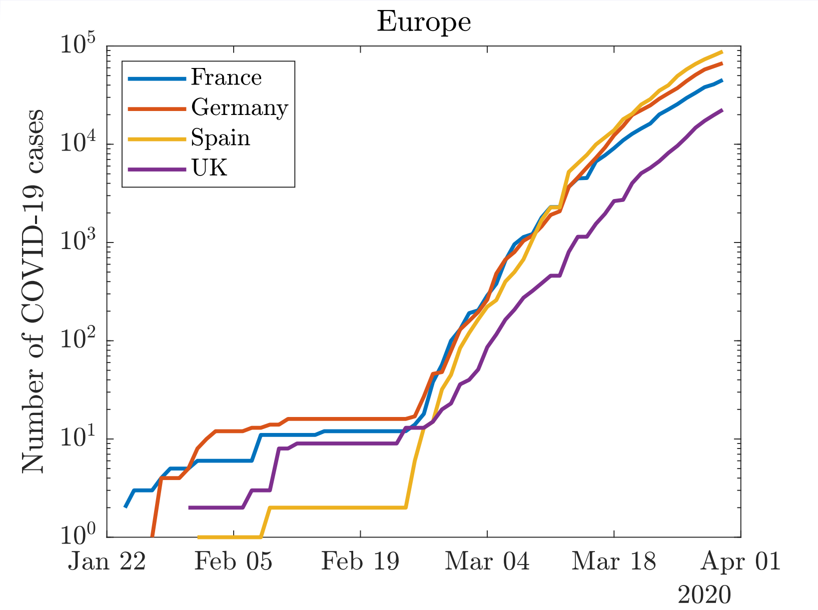
Policy implications

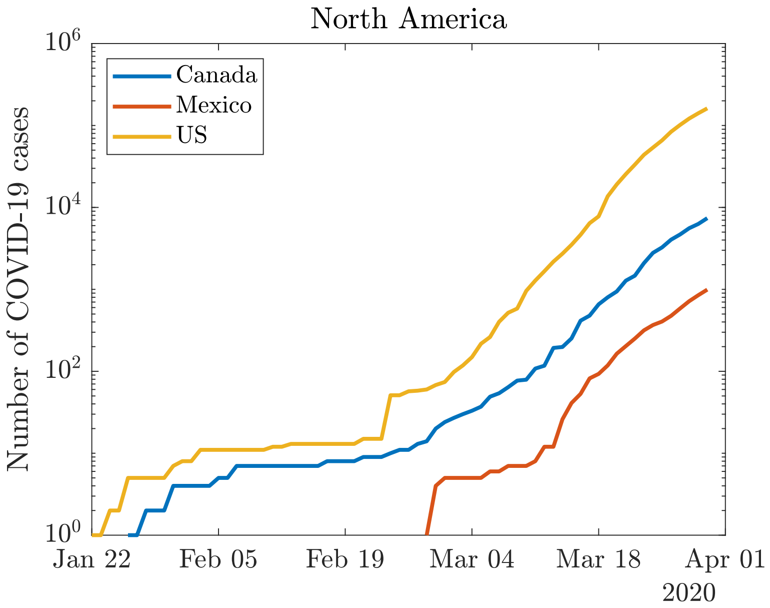
- Based on evidence from Spanish flu in 1918, mortality is higher during peak because healthcare system overwhelmed
- Hence important to reduce peak $y_{\max}$
- Because recovery rate γ is biological parameter that is out of control, can reduce peak by
  - Reduce susceptible x_0 (e.g., vaccination)
  - Reduce transmission rate β (e.g., social distancing, isolation)

Data

- Daily cases of COVID-19 provided by Center for Systems Science and Engineering (CSSE) at Johns Hopkins University
- Cumulative number of confirmed cases and deaths can be downloaded from GitHub repository ▸ link
- Time series starts on January 22, 2020 and updated daily
- Countries added as new cases reported
- For some countries (e.g., Australia, Canada, China), regional data available


Early epicenters
Number of COVID-19 cases
10^6
10^4
10^2
10^0
Jan 22
Feb 05
Feb 19
Mar 04
Mar 18
Apr 01
2020
China
Iran
Italy
Japan
Korea, South


Europe
France
Germany
Spain
UK
Number of COVID-19 cases
10^5
10^4
10^3
10^2
10^1
10^0
Jan 22
Feb 05
Feb 19
Mar 04
Mar 18
Apr 01
2020


North America
Canada
Mexico
US
Number of COVID-19 cases
10^6
10^4
10^2
10^0
Jan 22
Feb 05
Feb 19
Mar 04
Mar 18
Apr 01
2020

Estimation and prediction

- Basic idea: minimize loss function

$$L(\beta, \gamma, y_0, z_0) = \sum_t (\log \widehat{c}(t) - \log c(t))^2 ,$$

where $c, \widehat{c}$: fraction of cases in model and data

- Challenges:
  - COVID-19 situation (policies) rapidly evolving
  - Cases significantly underreported (∵ mild cases more likely undetected/untested)
  - recovery rate γ unidentified from cases
  - initial fraction of recovered (immune) z_0 unknown

Assumptions

- Use most recent 14 days of data (to deal with parameter changes)
- Testing/reporting rate constant during estimation period
- Set recovery rate $\gamma = 0.1$ assuming 10 days to recover as in influenza (14 days is more conservative)
- Estimate z_0 (initial fraction recovered) from death cases and reported mortality rate:

$$z_0 = \frac{\text{Number of deaths at } t = 0}{\text{Mortality}}$$

  (Still, actual z_0 likely much higher due to underreporting)
- For robustness, use median value of mortality among
  1. That country
  2. South Korea (most reliable data due to high testing rate)
  3. Global mortality

Estimation

- Basic idea: minimize loss function

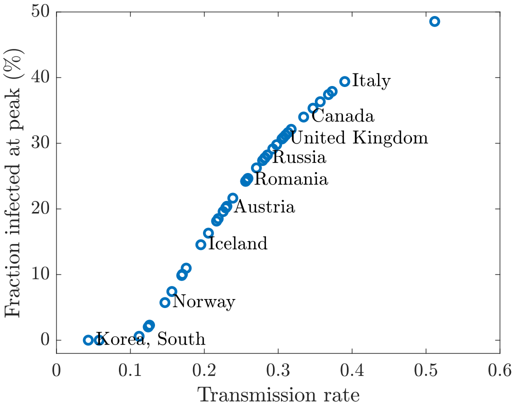
$$L(\beta, \gamma, y_0, z_0) = \sum_t (\log \widehat{c}(t) - \log c(t))^2 ,$$

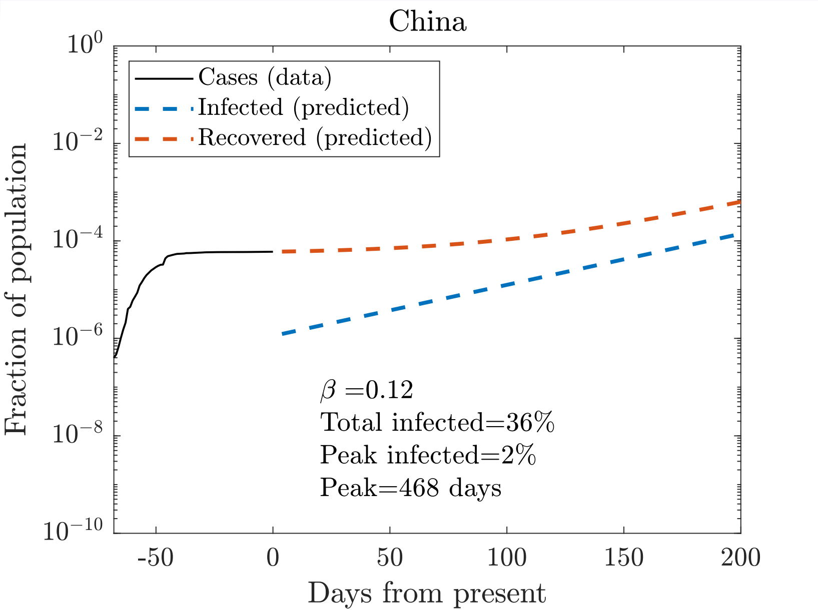
where $c, \widehat{c}$: fraction of cases in model and data

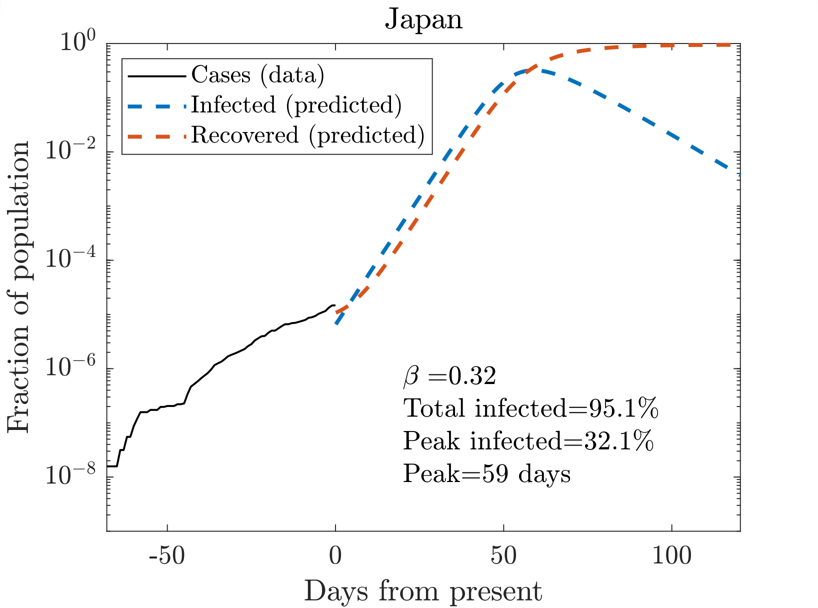
- As discussed, γ, z_0 are exogenously set
- Underreporting in cases absorbed by y_0, hence estimation of β likely consistent
- Interpretation of β: transmission rate during past 14 days (not natural rate)
- Compute standard errors using asymptotic theory of nonlinear least squares (M-estimator) and assuming independent measurement errors

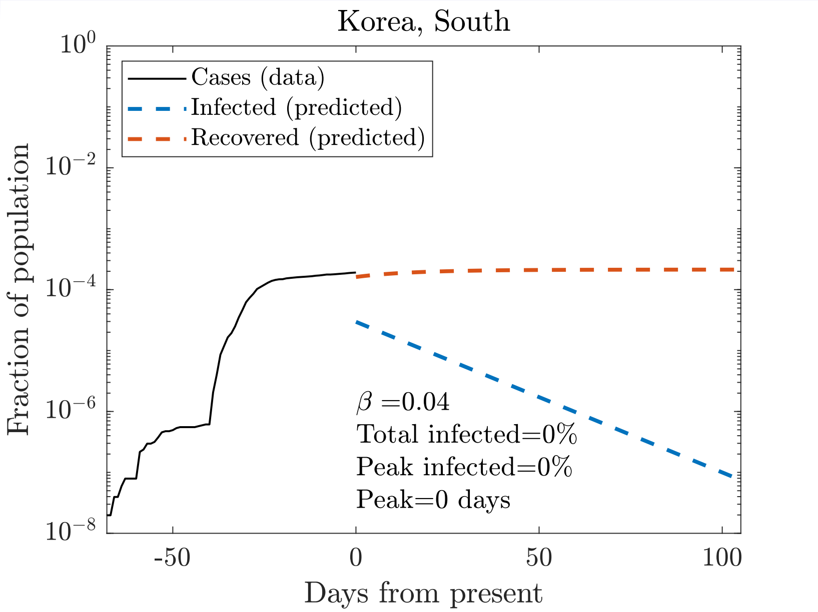
Results

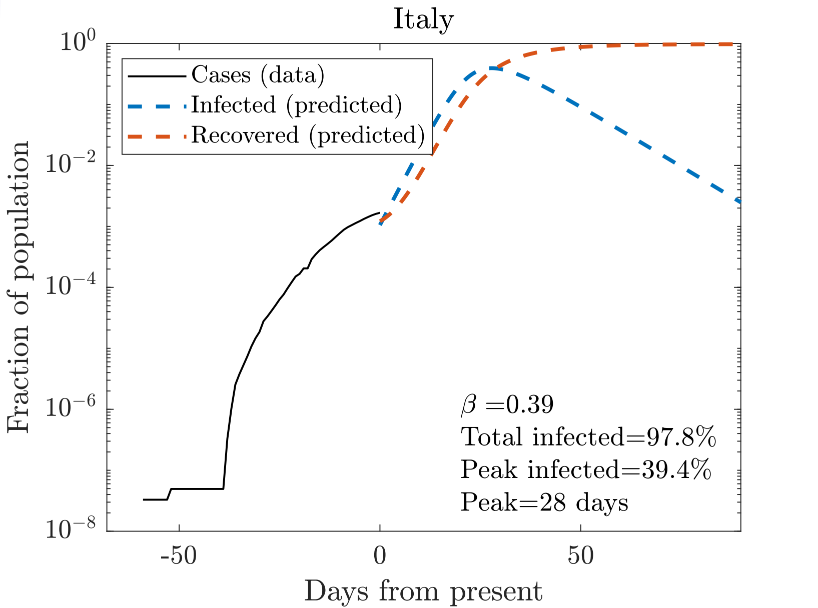
- Estimate SIR model for all countries with at least 1,000 cases now and 10 cases 14 days ago
- Median transmission rate: $\beta = 0.26$ (s.e. $\sim$ 0.01)
- Reproduction number $R = \beta/\gamma \sim 3$, so epidemic inevitable

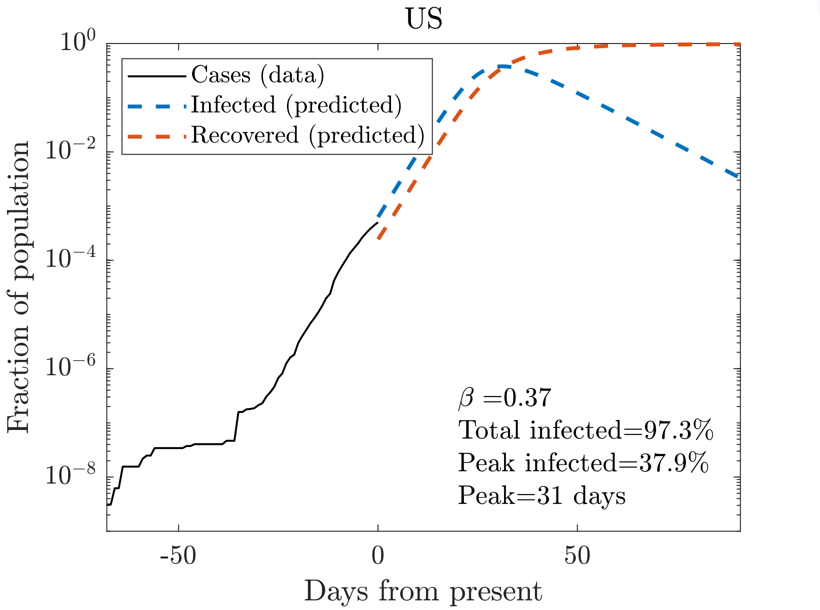


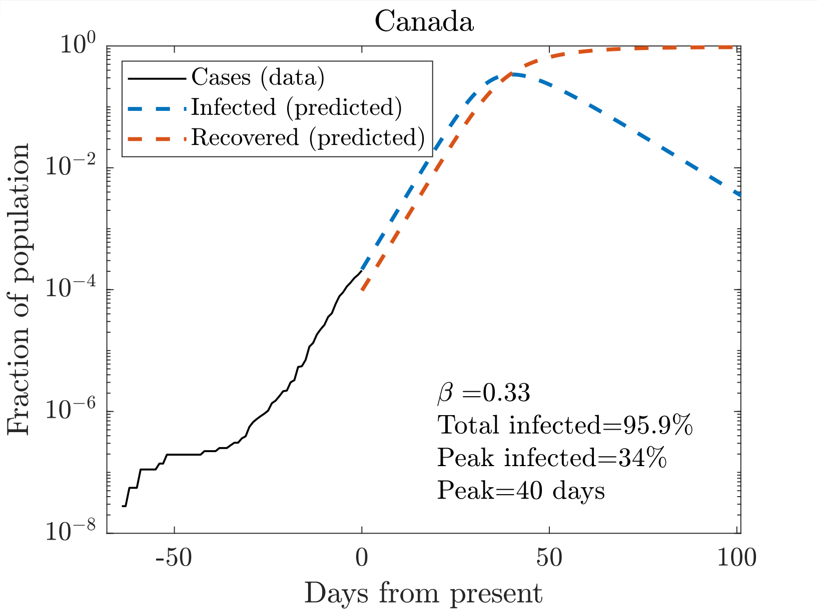

China
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.12
Total infected=36%
Peak infected=2%
Peak=468 days
Fraction of population
Days from present
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
-50
0
50
100
150
200

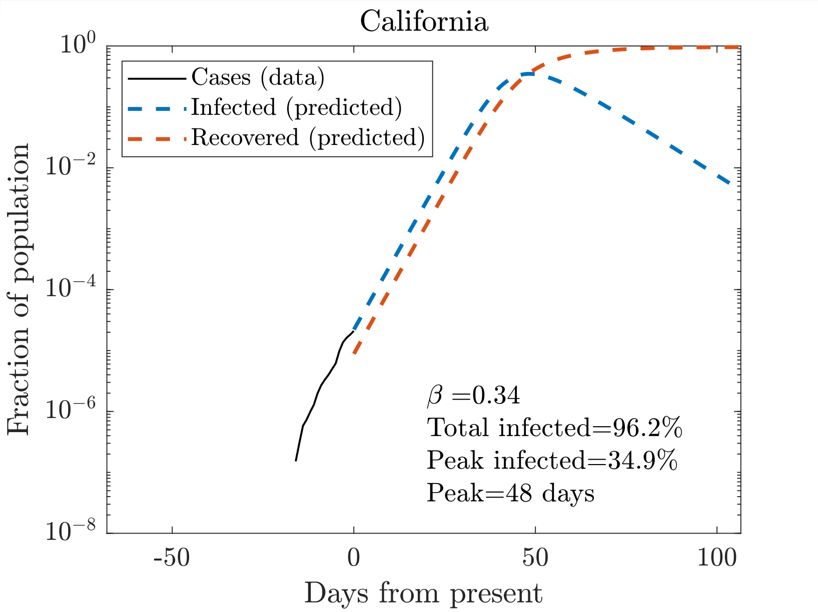

Japan
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.32
Total infected=95.1%
Peak infected=32.1%
Peak=59 days
Fraction of population
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
-50
0
50
100
Days from present


Korea, South
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
Fraction of population
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.04
Total infected=0%
Peak infected=0%
Peak=0 days
-50
0
50
100
Days from present


Italy
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.39
Total infected=97.8%
Peak infected=39.4%
Peak=28 days
Fraction of population
Days from present
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
-50
0
50


US
Fraction of population
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
-50
0
50
Days from present
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.37
Total infected=97.3%
Peak infected=37.9%
Peak=31 days


Canada
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.33
Total infected=95.9%
Peak infected=34%
Peak=40 days
Fraction of population
Days from present
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
-50
0
50
100

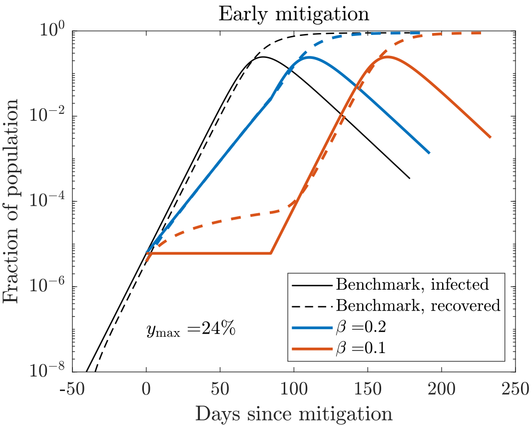

California
Cases (data)
Infected (predicted)
Recovered (predicted)
β =0.34
Total infected=96.2%
Peak infected=34.9%
Peak=48 days
Fraction of population
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
-50
0
50
100
Days from present

Mitigation policies

- Suppose government takes mitigation measures (e.g., social-distancing, restricting travel, closing schools, etc.)
- Reduce transmission rate β to 0.2 or 0.1
- Start mitigation measures when fraction of cases c reaches 10^{-5}
- Assume mitigation measures lifted after 12 weeks

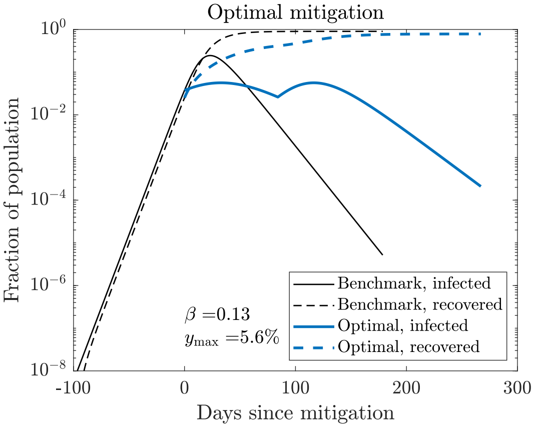
Suboptimality of early mitigation

- Early mitigation delays peak but size unaffected
- Intuition: prevents building herd immunity



Optimal mitigation

- Assume government can control timing and intensity of mitigation measures and minimize peak infection rate $y_{\max}$
- By optimally delaying intervention, can reduce peak
- Intuition: build herd immunity (low x)



Asset pricing with epidemic

- Consider stylized production-based asset pricing model
- Evaluate effect of epidemic on stock price under
  1. laissez-faire (no intervention)
  2. optimal mitigation

Model

- Capitalists and workers
  - Capitalists: hold capital, trade financial assets, CRRA preferences
  - Workers: supply labor and hand-to-mouth
- Capital exogenous and i.i.d. growth: $\log K_{t+1}/K_t \sim N(\mu, \sigma^2)$
  - Can interpret as Solow growth model with TFP growth
- Production Cobb-Douglas: $Y = K^\alpha L^{1-\alpha}$
- Infected workers can't supply labor
- Stock is claim to profit $K^\alpha L^{1-\alpha} - wL$

Solution

- By market clearing and profit maximization,

$$C = D = K^\alpha L^{1-\alpha} - wL = \alpha K^\alpha L^{1-\alpha}$$

- No-arbitrage condition:

$$P_t = \mathrm{E}_t \left[\beta (C_{t+1}/C_t)^{-\gamma} (P_{t+1} + C_{t+1})\right]$$

- Hence price-dividend ratio satisfies

$$V_t = \mathrm{E}_t \left[\beta \left((K_{t+1}/K_t)^\alpha (L_{t+1}/L_t)^{1-\alpha}\right)^{1-\gamma} (V_{t+1} + 1)\right]$$

- Using lognormal capital growth, get

$$V_t = \kappa (L_{t+1}/L_t)^{(1-\alpha)(1-\gamma)} (V_{t+1} + 1),$$

where $\kappa = \beta \mathrm{e}^{\alpha(1-\gamma)\mu + [\alpha(1-\gamma)]^2 \sigma^2/2}$

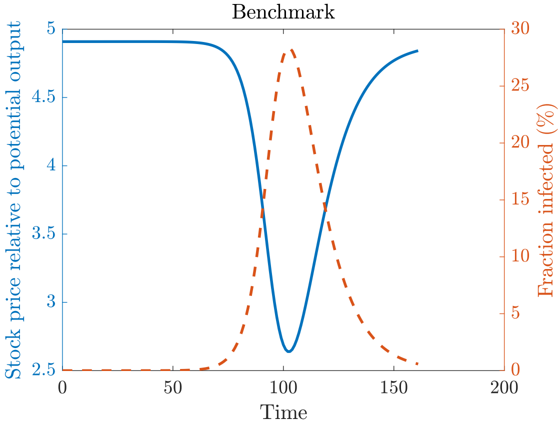
- Straightforward to numerically solve by iteration

Calibration

- $\alpha = 0.38$, $\gamma = 3$, $\beta = 4\%$ discount (standard)
- Capital growth and volatility calibrated from GDP growth
    - In steady state model, $Y = K^\alpha$, so $\Delta \log Y = \alpha \Delta \log K$
- Transmission rate 0.3 (median value)
- Recovery rate 0.1 (10 days)
- Initial condition $(y_0, z_0) = (10^{-8}, 0)$ (not important)

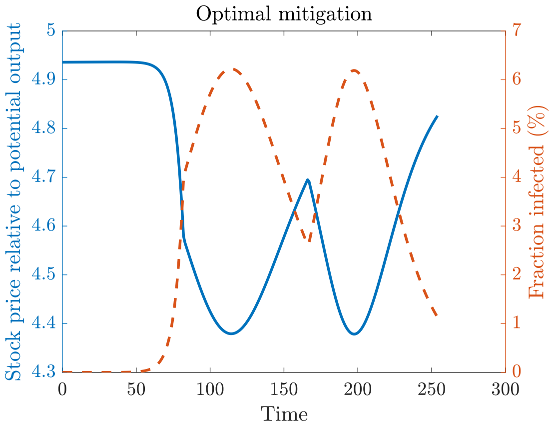
Stock price during epidemic

- Without mitigation, stock price crashes by 50%
- Crash short-lived (temporary supply shock)



Stock price with optimal mitigation

- With optimal mitigation, stock price decreases by 10%
- Bear market lasts for half a year, with two troughs



Conclusion

- COVID-19 situation rapidly evolving, so any analysis based on current data will quickly become out of date, but
  1. Now that community transmission is widespread and $\beta \approx 0.3 \gg 0.1 \approx \gamma$, epidemic likely inevitable
  2. (Too) early mitigation buys time (to develop vaccine, build respirators, masks, & gloves, and train people), but no effect on peak (unless prolonged mitigation)
  3. Optimally delayed mitigation may reduce peak

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  3. Optimally delayed mitigation may reduce peak
- My view is that current lock-down measures in California are too early & extreme; unsustainable drastic measures suboptimal
- Maybe targeted isolation of elderly & those with underlying conditions is better

Diamond Princess

- In February 2020, cruise ship Diamond Princess put under quarantine after COVID-19 detected
- All passengers tested and tracked
- 712 tested positive and 8 died; hence mortality $= 8/712 \approx 1\%$
- Cruise ship passengers older than general population, so likely overestimate ▸ Return

2003 SARS

- In 2002–2004, there was outbreak of severe acute respiratory syndrome (SARS) caused by another coronavirus
- 8,096 cases and 775 deaths globally (9.6%), quickly contained by isolation of patients and contact tracking
- COVID-19 seems much more contagious but much less lethal than SARS ▸ Return