

Optimal Epidemic Control in Equilibrium with Imperfect Testing and Enforcement*

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*The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System.

COVID-19 epidemic

- ▶ Started in December 2019 in Wuhan, China
- ▶ First community spread outside China and Korea reported in late February 2020 in Italy
- ▶ European countries quickly implemented mitigation policies such as lockdown
- ▶ U.S., Canada, and other countries followed suit by mid-March

Are mitigation policies warranted?

- ▶ At first glance, protecting public from infectious disease appears to be a responsible government action

Are mitigation policies warranted?

- ▶ At first glance, protecting public from infectious disease appears to be a responsible government action
- ▶ After second thought, draconian social distancing policies no longer appear self-evident
 1. Closure of schools and businesses have obvious economic costs
 2. Social distancing prevents new infections but also building *herd immunity*—hence policies may never achieve goal of ending epidemic [▶ VoxEU column](#)
 3. Mathematical epidemic models used in policy-making ignore forward-looking behavior by rational agents
- ▶ Calls for rigorous theoretical (and quantitative) analysis

This paper

- ▶ Theoretically studies Susceptible-Infected-Recovered (SIR) epidemic model with forward-looking, rational agents
- ▶ Features:
 - ▶ Agents choose economic activity level optimally, understanding infection risk
 - ▶ Cases may be underreported and agents need to infer their health status (empirically, only 20–40% of cases diagnosed)
 - ▶ Government may have limited power to enforce optimal policy

Main results (theoretical)

1. Prove existence of perfect Bayesian Markov competitive equilibrium
 - ▶ Here
 - ▶ “Perfect Bayesian”: agents use Bayes rule on equilibrium path
 - ▶ “Markov”: policy functions depend on state variables
 - ▶ “Competitive”: agents ignore effects of their behavior on aggregate variables
 - ▶ Markov structure important because we can talk about optimal policy at any point in time or point in state space
2. Prove approximate static efficiency of equilibrium
 - ▶ Equilibrium inefficient due to externalities
 - ▶ Static: infected agents infect others
 - ▶ Dynamic: collective behavior of agents affects future dynamics
 - ▶ Static externality by unknown infected agents small

Main results (quantitative)

- ▶ Numerically solve for equilibrium and optimal policy of model of COVID-19 epidemic
- ▶ Findings:
 - ▶ Endogenous social distancing by agents mitigates epidemic, but welfare gain modest ($\sim 10\%$ reduction in welfare costs)
 - ▶ Additional welfare gain from lockdown policy small ($< 10\%$)
 - ▶ Quarantine effective, even with imperfect testing

Literature

- ▶ **Mathematical epidemic model:** Kermack & McKendrick (1927)
- ▶ **Non-strategic economic epidemic model:** Sethi (1978), Kruse & Strack (2020)
- ▶ **Strategic epidemic model:**
 - ▶ (HIV) Geoffard & Philipson (1996), Kremer (1996), Auld (2003)
 - ▶ (Community infectious disease) Reluga (2010), Chen (2012), Fenichel (2013), **Toxvaerd (2020)**
- ▶ **Failure of well-intended public health policy** Toxvaerd (2019)

Society, agent types

- ▶ Agents: $n = 1, \dots, N$ (finite but large)
- ▶ Time: $t = 0, \Delta, 2\Delta, \dots$ (discrete, infinite horizon)
- ▶ Agent types by health/information
 - ▶ S : susceptible (no immunity)
 - ▶ I_k : known infected (diagnosed)
 - ▶ I_u : unknown infected (undiagnosed)
 - ▶ R_k : known recovered (diagnosed)
 - ▶ R_u : unknown recovered (undiagnosed)
 - ▶ D : dead
- ▶ Behavioral types are denoted by $h \in \{U, I_k, R_k, D\}$, where $U = S \cup I_u \cup R_u$ is **unknown** type

State variables

- ▶ Individual state variables are health/information status
 $h \in \{U, I_k, R_k, D\}$
- ▶ Aggregate state variables are fraction of each agent type
 - ▶ Let N_h be number of type $h \in \{S, I_k, I_u, R_k, R_u, D\}$ agents
 - ▶ With slight abuse of notation, let $h = N_h/N$ be fraction of type h agents
 - ▶ Then state space is set Z of $z = (S, I_k, I_u, R_k, R_u, D)$, where

$$S + I_k + I_u + R_k + R_u + D = 1,$$

$$N(S, I_k, I_u, R_k, R_u, D) \in \mathbb{Z}_+^6$$

- ▶ Assume aggregate state observable (can be inferred from random testing)

Preferences

- ▶ Alive agents (excluding I_k) take action $a \in A = [\underline{a}, 1] \subset [0, 1]$ with flow payoff $u(a)$, where $u(1) = 0$, $u' > 0$, $u'' < 0$
 - ▶ Interpretation: a is “economic activity level”, with $a = 1$ being normal life and $a = \underline{a}$ being locked down
- ▶ Known infected (I_k) agents have utility function $u_I : A \rightarrow \mathbb{R}$, single-peaked at $a_I \in A$
- ▶ Dead agents receive flow utility $u_D < 0$, where $u_D \leq u_I(a) \leq u(a)$ for all $a \in A$
- ▶ Agents maximize expected discounted payoffs at rate $e^{-r\Delta}$, where $r > 0$: discount rate

Disease transmission

- ▶ Agents meet randomly and transmit infectious disease
- ▶ If n, n' take actions $a_n, a_{n'}$, then they meet with probability $\lambda \Delta a_n a_{n'} / N$, where λ : meeting rate
 - ▶ λ exogenous and depends on how society is organized, e.g., population density, commuting, shopping, teaching pattern
- ▶ Conditional on n meeting n' and n' being infected, n gets infected with probability τ
 - ▶ τ exogenous and depends on contagiousness and society organization (greet by bowing, shaking hands, hugging, kissing)
- ▶ New infection diagnosed with probability $\sigma \in (0, 1]$
- ▶ If type h agents take average action a_h , then infection probability

$$\Pr(\text{get infected} \mid a, \text{susceptible}) = \beta \Delta (a_{I_k} I_k + a_{U I_u}) a,$$

where $\beta = \tau \lambda$: transmission rate

Recovery, death, and vaccine arrival

- ▶ Every period, infected agents removed (recover or die) with probability $\gamma\Delta$
- ▶ I_u agents always recover; conditional on removal, I_k agents die with probability δ
 - ▶ δ : **Case Fatality Rate (CFR)** (fatality rate among reported cases)
 - ▶ $\delta_0 := \sigma\delta$: **Infection Fatality Rate (IFR)** (fatality rate among all cases)
- ▶ Recovered and dead agents remain so forever (lifelong immunity)
- ▶ Vaccine arrives at Poisson rate ν ; once vaccine arrives, all alive non-infected agents become R_k

Mathematical epidemic models

- ▶ Our framework is basic SIR(D) model
- ▶ Other variations:
 - ▶ **SI**: infected agents remain infected forever (e.g., Epstein-Barr virus infection)
 - ▶ **SIS**: infected agents recover but can be reinfected (e.g., seasonal influenza)
 - ▶ **SEIR**: exposed agents are infected but not yet contagious
- ▶ For COVID-19, unclear whether immunity is lifelong
 - ▶ Probably SIRDS (possibility of $R \rightarrow S$ as in influenza) more appropriate, but in short run SIRD should be enough

Assumptions

Assumption (Perfect competition)

Agents view the evolution of the aggregate state z as exogenous and ignore the impact of their behavior on the aggregate state.

Assumption (Consistency)

On equilibrium paths, agents update their beliefs using the Bayes rule. Off equilibrium paths, unknown (U) agents believe they are susceptible with probability

$$\mu(z) := \begin{cases} \frac{S}{S+I_u+R_u} & \text{if } S > 0, \\ 0 & \text{otherwise.} \end{cases}$$

▶ Skip equilibrium analysis

Individual problems: known recovered and dead

- ▶ Let $V_h(z)$ be value function of type h agents
- ▶ Because dead agents remain dead, Bellman equation is

$$V_D = (1 - e^{-r\Delta})u_D + e^{-r\Delta}V_D \iff V_D = u_D.$$

- ▶ Because known recovered agents remain recovered, Bellman equation is

$$V_{R_k} = \max_{a \in A} \left\{ (1 - e^{-r\Delta})u(a) + e^{-r\Delta}V_{R_k} \right\}.$$

- ▶ Optimal policy is clearly $a_{R_k} = 1$ and value function is $V_{R_k} = 0$

Individual problems: known infected

- ▶ Known infected agents are removed with probability $\gamma\Delta$, and conditional on removal, die with probability $\delta = \delta_0/\sigma$
- ▶ Hence Bellman equation is

$$V_{I_k} = \max_{a \in A} \left\{ (1 - e^{-r\Delta}) u_I(a) + e^{-r\Delta} \left(\underbrace{(1 - \gamma\Delta) V_{I_k}}_{\text{stay infected}} + \underbrace{\gamma\Delta [(1 - \delta) V_{R_k} + \delta V_D]}_{\text{removal}} \right) \right\}.$$

- ▶ Hence optimal policy is $a_{I_k} = a_I$ and value function is

$$V_{I_k} = \frac{(1 - e^{-r\Delta}) u_I + e^{-r\Delta} \gamma \Delta \delta u_D}{1 - e^{-r\Delta} (1 - \gamma \Delta)},$$

where $u_I := u_I(a_I)$

Individual problems: unknown

- ▶ Suppose unknown agents adhere to policy function $a_U(z)$ and have belief $\mu(z)$ above
- ▶ Let $p(z) = \beta\Delta(a_I I_k + a_U(z) I_u)$ be probability of infection with full action ($a = 1$)
- ▶ Then Bellman equation is

$$\begin{aligned}
 V_U(z) = \max_{a \in A} & \left\{ (1 - e^{-r\Delta})u(a) + \underbrace{e^{-r\Delta}\sigma\mu pa V_{I_k}}_{\text{known infection}} \right. \\
 & \left. + \underbrace{e^{-r\Delta}(1 - \sigma\mu pa)}_{\text{stay unknown}} E_z \left(\underbrace{e^{-\nu\Delta} V_U(z')}_{\text{no vaccine}} + \underbrace{(1 - e^{-\nu\Delta}) V_{R_k}}_{\text{vaccine}} \right) \right\}
 \end{aligned}$$

Value functions

Proposition (Value functions)

Fix a policy function $a_U : Z \rightarrow A$ of unknown agents. Then there exists a unique value function $V_U : Z \rightarrow \mathbb{R}$ satisfying the Bellman equation. Furthermore,

$$\begin{aligned}
 V_D = u_D &< \frac{(e^{r\Delta} - 1)u_I + \gamma\Delta\delta u_D}{e^{r\Delta} - 1 + \gamma\Delta} = V_{I_k} \\
 &< \frac{e^{\nu\Delta}\sigma\beta\Delta}{e^{(r+\nu)\Delta} - 1 + \sigma\beta\Delta} V_{I_k} \leq V_U(z) \leq V_{R_k} = 0.
 \end{aligned}$$

- ▶ $V_D \leq V_{I_k}$ and $V_U \leq V_{R_k}$ obvious
- ▶ $V_{I_k} \leq V_U$ because U agents can always take $a = 1$ and $u_I \leq u$

Best response of U agents

Proposition (Best response of U agents)

Fix a policy function $a_U : Z \rightarrow A$ and a continuation value $V_U : Z \rightarrow \mathbb{R}$ of unknown agents. Then the best response of an unknown agent is

$$a^* = \phi \left(\frac{\sigma \mu(z) p(z)}{e^{r\Delta} - 1} (E_z e^{-\nu\Delta} V_U(z') - V_{I_k}) \right),$$

where

$$\phi(x) := \begin{cases} 1 & \text{if } x \leq u'(1), \\ (u')^{-1}(x) & \text{if } u'(1) < x < u'(\underline{a}), \\ \underline{a} & \text{if } x \geq u'(\underline{a}). \end{cases}$$

is inverse marginal utility function.

Comparative statics

- ▶ Recall best response is

$$a^* = \phi \left(\frac{\sigma \mu(z) p(z)}{e^{r\Delta} - 1} (\mathbb{E}_z e^{-\nu \Delta} V_U(z') - V_{I_k}) \right)$$

- ▶ Because $u'' < 0$, ϕ is decreasing, so a^* decreasing in ν, μ, p, δ_0 and increasing in u_I, u_D
- ▶ So U agents take higher action if vaccine arrival less likely, infection less likely, death less likely, and infection and death less scary

Equilibrium

- ▶ Our equilibrium concept is *perfect Bayesian Markov competitive equilibrium*
 - ▶ “Perfect Bayesian”: agents use Bayes rule on equilibrium path
 - ▶ “Markov”: policy functions depend on state variables
 - ▶ “Competitive”: agents ignore effects of their behavior on aggregate variables
- ▶ Justification of competitive behavior: N large

Perfect Bayesian Markov competitive equilibrium

Definition (Markov equilibrium)

A (pure strategy) *perfect Bayesian Markov competitive equilibrium* consists of unknown agents' belief $\mu(z)$ of being susceptible, transition probabilities $\{q(z, z')\}_{z, z' \in Z}$ for aggregate state, value functions $\{V_h(z)\}_{h=U, I_k, R_k, D}$, and policy functions $\{a_h(z)\}_{h=U, I_k, R_k}$ such that

1. (Consistency) The belief $\mu(z)$ satisfies the Bayes rule on equilibrium paths; the transition probabilities $\{q(z, z')\}$ are consistent with individual actions and the mechanisms of disease transmission, symptom development, recovery, and death,
2. (Sequential rationality) Bellman equations hold

Existence of equilibrium

Theorem (Existence of equilibrium)

Under maintained assumptions, there exists a pure strategy perfect Bayesian Markov competitive equilibrium, where the belief $\mu(z)$ always satisfies

$$\mu(z) := \begin{cases} \frac{S}{S+I_u+R_u} & \text{if } S > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Idea of proof

- ▶ Let $a_U(z)$ be a guess of policy function and $V_U(z)$ associated value function; note state space Z is finite, so $(a_U(z))_{z \in Z}$ is finite-dimensional
- ▶ Update $a_U(z)$ using best response function
- ▶ Can show continuity of this map ▶ Lemma
- ▶ Apply Brouwer fixed point theorem

Externalities and efficiency

- ▶ Equilibrium is inefficient due to two types of externalities
 - ▶ **Static:** when infected agents go out, they infect others in *same* period
 - ▶ **Dynamic:** agents' collective behavior affects dynamics of *future* state variables
- ▶ Dynamic externality is subtle: e.g., when agents stay home, new infections reduced but epidemic prolonged
- ▶ Theoretically studying dynamic externality challenging, hence focus on static externality

Social welfare function

- ▶ In state $z \in Z$, utilitarian social welfare is

$$\mathcal{W}(z) := (S + I_u + R_u)V_U(z) + I_k V_{I_k}(z) + R_k V_{R_k}(z) + DV_D(z)$$

- ▶ Since agents' behavior does not affect V_{R_k} and V_D , suffices to consider

$$W(z) := UV_U(z) + I_k V_{I_k}(z)$$

Lockdown and quarantine policies

- ▶ Since interested in eliminating static externality, take continuation value V_U, V_{I_k} and transition probability $q(z, z')$ as given and maximize

$$W(a_U, a_{I_k}, z) := U \left[(1 - e^{-r\Delta}) u(a_U) + e^{-r\Delta} E_z \left((1 - \sigma \mu p a_U) e^{-\nu \Delta} V_U(z') + \sigma \mu p a_U V_{I_k}(z') \right) \right] + I_k \left[(1 - e^{-r\Delta}) u_I(a_{I_k}) + e^{-r\Delta} E_z \left((1 - \gamma \Delta) V_{I_k}(z') + \gamma \Delta \delta V_D \right) \right].$$

- ▶ We refer to a_U and a_{I_k} chosen by planner as **lockdown** and **quarantine** policies
- ▶ Lockdown policy a_U^\dagger is **static efficient** if it maximizes W given a_{I_k}

Approximate static constrained efficiency of equilibrium

Theorem (Static efficient lockdown policy)

Let $a_U^*(z)$ be the equilibrium policy of unknown agents and $m = \min_{a \in A} |u''(a)| > 0$. There exists a unique static efficient lockdown policy $a_U^\dagger(z)$, which satisfies

$$\begin{aligned} 0 &\leq a_U^*(z) - a_U^\dagger(z) \\ &\leq \frac{\sigma\beta\Delta}{m(e^{r\Delta} - 1)} \mu(z) a_U^*(z) I_U E_z(e^{-\nu\Delta} V_U(z') - V_{I_k}). \end{aligned}$$

In particular, if $\sigma = 1$ (so $I_U = 0$), then $a_U^\dagger(z) = a_U^*(z)$.

Intuition of theorem

- ▶ Proof is long and technical; uses fact that if f is monotone and f, g are close, then roots of f and g are close [▶ Details](#)
- ▶ Inequality $a_U^* \geq a_U^\dagger$ comes from fact that unknown agents are not internalizing negative externality caused by unknown infected agents
- ▶ Individual agents view utility loss from infection as linear function of own activity, while planner views as quadratic due to interaction between susceptible and unknown infected agents
- ▶ Upper bound of $a_U^* - a_U^\dagger$ comes from similarity of individual and planner first-order conditions

Epidemic dynamics

- ▶ Fix any policy functions $(a_U(z), a_{I_k}(z))$ and induced transition probabilities $\{q(z, z')\}$,
- ▶ Then

$$E_z(S_{t+\Delta} - S_t)/\Delta = -\beta a_U(z)(\sigma a_{I_k}(z) + (1 - \sigma)a_U(z))S_t I_t,$$

$$E_z(I_{t+\Delta} - I_t)/\Delta = (\beta a_U(z)(\sigma a_{I_k}(z) + (1 - \sigma)a_U(z))S_t - \gamma)I_t,$$

$$E_z(R_{t+\Delta} - R_t)/\Delta = \gamma(1 - \delta)I_t,$$

$$E_z(D_{t+\Delta} - D_t)/\Delta = \gamma\delta I_t.$$

- ▶ Standard SIR(D) model special case by letting $a_U = a_{I_k} = 1$, $N \rightarrow \infty$, $\Delta \rightarrow 0$
- ▶ Definition: *herd immunity* achieved if $\mathcal{R}_0 S_t \leq 1$ (where $\mathcal{R}_0 = \beta/\gamma$: reproduction number), implying $\dot{I}_t \leq 0$

Model specification

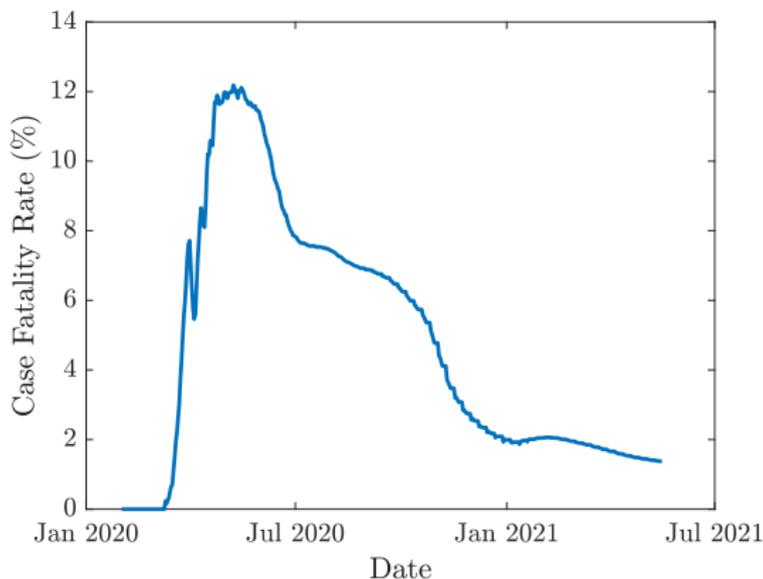
- ▶ One period is a day
- ▶ Annual 5% discounting, so $r = 0.05/365.25$
- ▶ Vaccine arrives in one year, so $\nu = 1/365.25$
- ▶ Daily transmission rate $\beta = 1/5.4$, from meta-analysis of Rai et.al. (2021)
- ▶ Daily recovery rate $\gamma = 1/13.5$, from You et.al. (2020), so basic reproduction number $\mathcal{R}_0 = \beta/\gamma = 2.5$
- ▶ Infection fatality rate $\delta_0 = 0.0027$, from meta-analysis of Ioannidis (2021)
- ▶ Diagnosis rate $\sigma = 0.4$ (Empirical case fatality rate $\delta = 0.0133$ implies $\sigma = 0.2$; whereas Diamond Princess implies symptom 50%)

Model specification

- ▶ Log utility, so $u(a) = \log a$
- ▶ Assume $u_I(a) = u(a)$, so optimal action of I_k agents $a_I = 1$ (worst case analysis)
- ▶ Calibrate $u_D = -12.22$ from Sweden data, which did not introduce lockdown
- ▶ Initial condition: $I_0 = 10^{-6}$, $S_0 = 1 - I_0$, $R_0 = 0$, $D_0 = 0$

Case fatality rate in Sweden

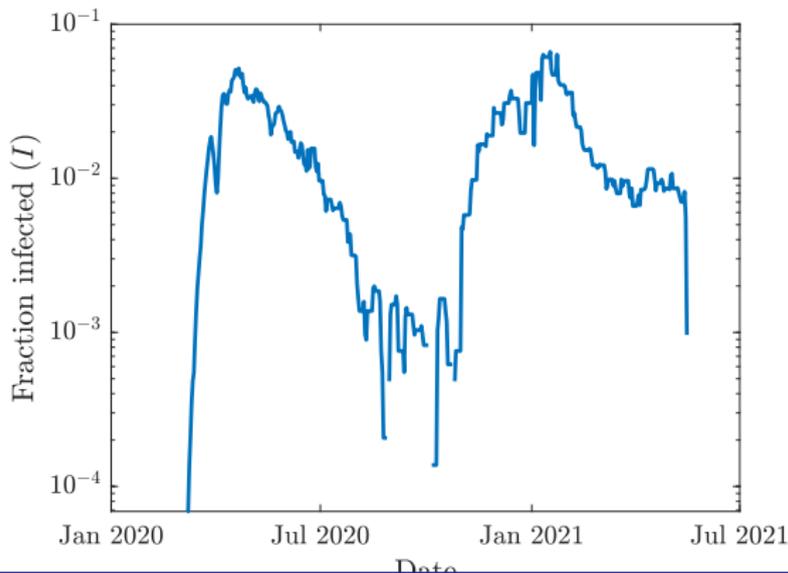
- ▶ $\delta_{\text{CFR}} = 0.0158$, so reporting rate $\sigma = 0.171$ in Sweden



Prevalence in Sweden

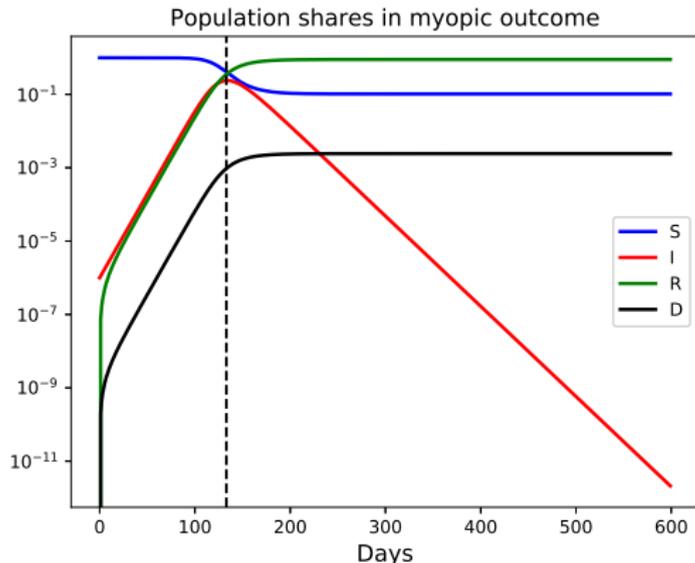
- ▶ To back up prevalence, use accounting equation

$$D_{t+1} - D_t = \gamma \delta_{\text{IFR}} I_t \iff I_t = (D_{t+1} - D_t) / \gamma \delta_{\text{IFR}}$$



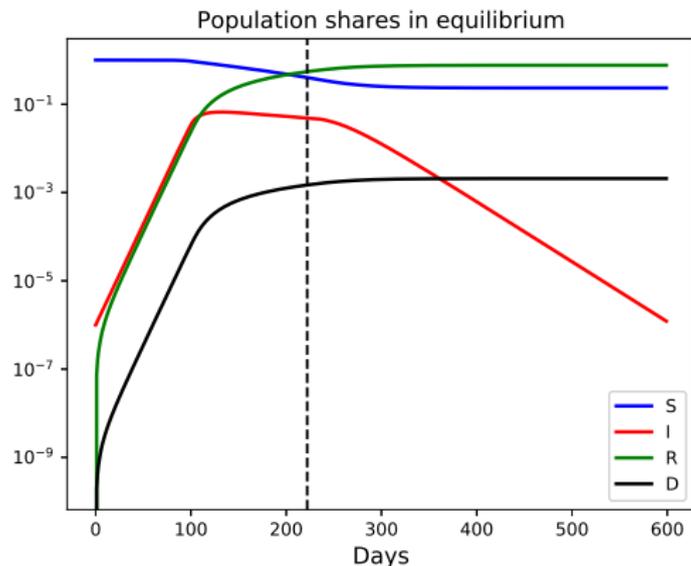
Myopic equilibrium

- Agents choose myopic optimal action $a = 1$ (standard SIR model)

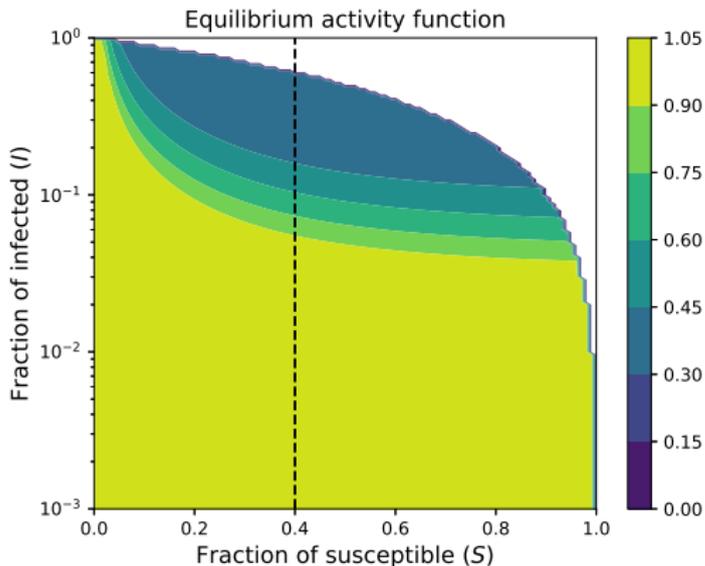


Markov equilibrium

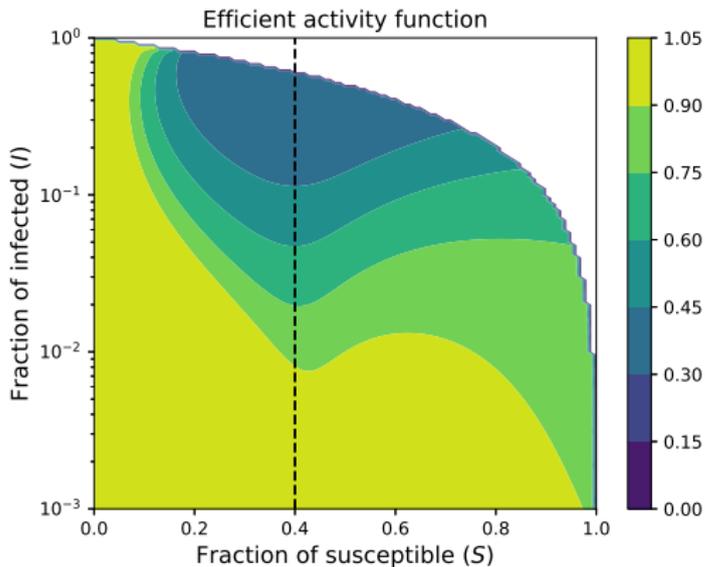
- Agents choose individually optimal actions $a_U(z)$, $a_{I_k}(z)$



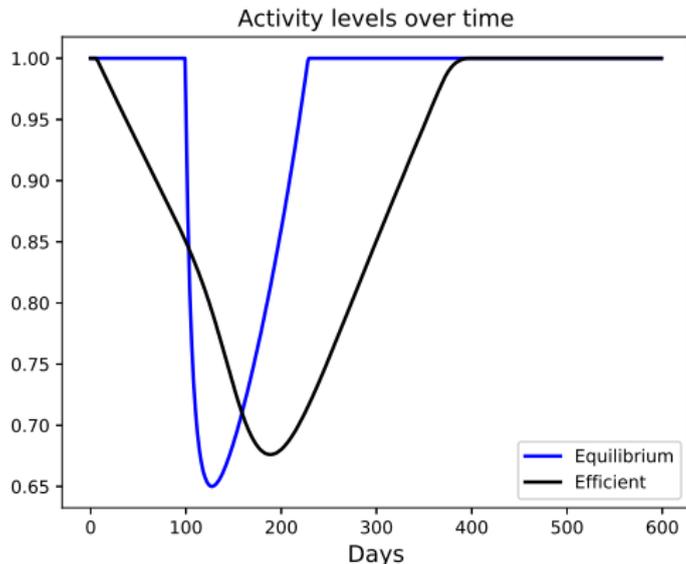
Equilibrium action of unknown agents



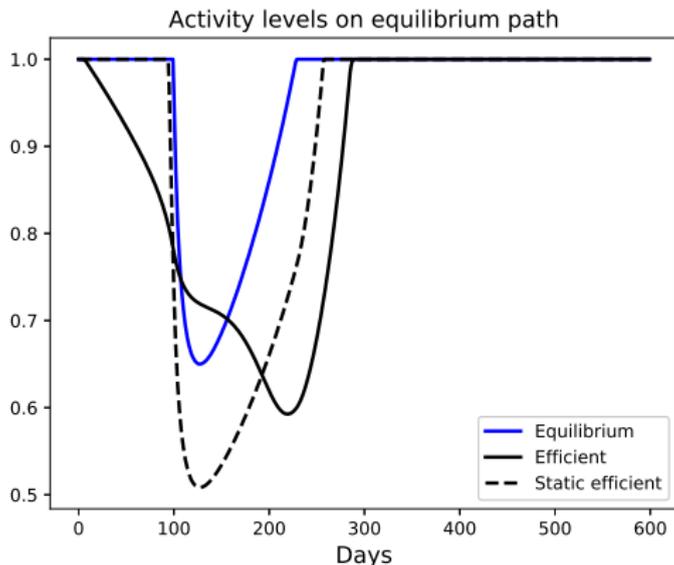
Efficient action of unknown agents



Recommended action over time



Recommended action over equilibrium path



Welfare cost and death toll

- ▶ σ : diagnosis rate
- ▶ Welfare gains from lockdown quite modest

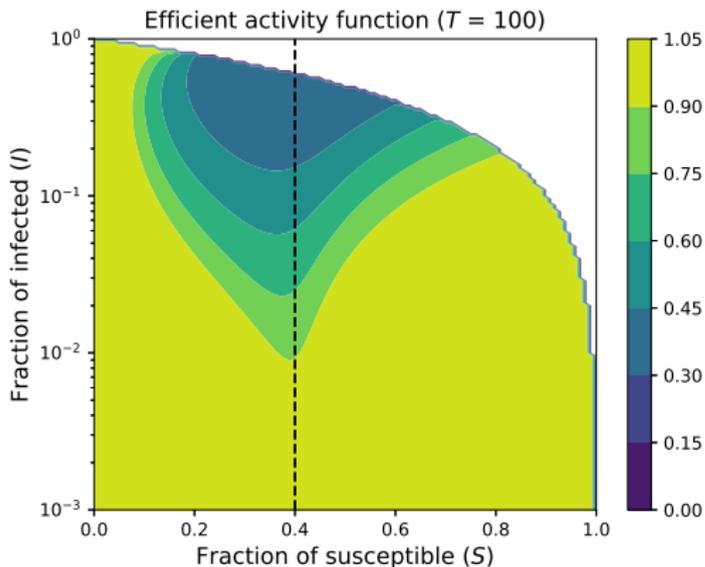
σ	Welfare loss (%)			Death toll (per 100,000)		
	Myopic	PBE	SPP	Myopic	PBE	SPP
0.1	2.04	1.80	1.68	242	209	172
0.2	2.04	1.81	1.70	242	208	175
0.4	2.04	1.82	1.74	242	207	178
0.7	2.03	1.83	1.76	242	204	179
1.0	2.04	1.81	1.76	242	201	178

Importance of quarantine

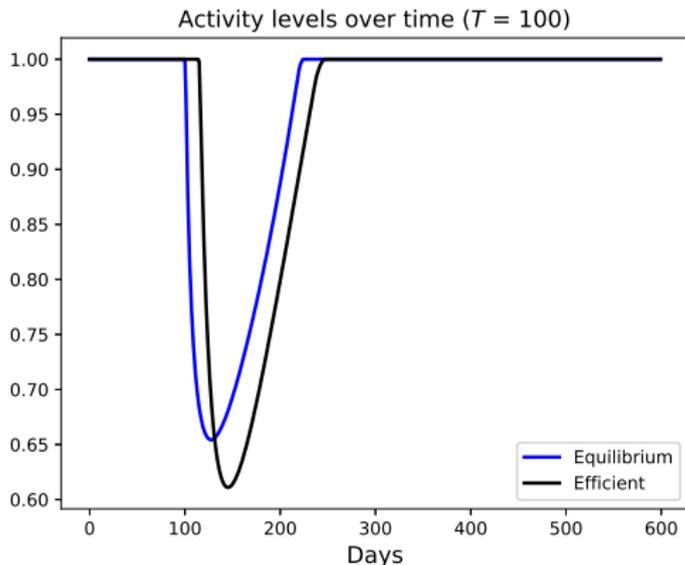
- ▶ So far we assume infected agents choose $a_I = 1$
- ▶ Suppose known infected agents are quarantined at $a_I = 0.4$ (He et al. (2020) document that 44% of secondary cases were infected prior to diagnosis of primary cases)

σ	Welfare loss (%)			Death toll (per 100,000)		
	Myopic	PBE	SPP	Myopic	PBE	SPP
0.1	1.94	1.69	1.53	236	204	171
0.2	1.80	1.57	1.41	229	197	163
0.4	1.45	1.28	1.11	208	182	143
0.7	0.65	0.64	0.52	149	148	102
1.0	0.00	0.00	0.00	0	0	0

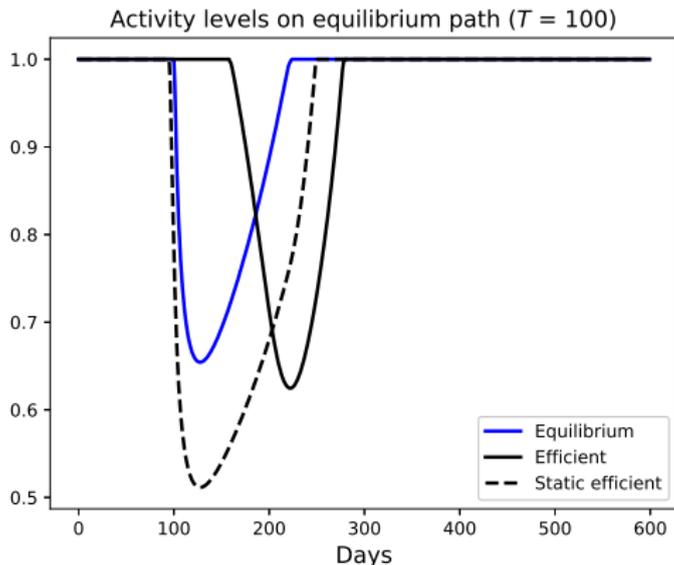
Efficient action without vaccine



Recommended action over time without vaccine



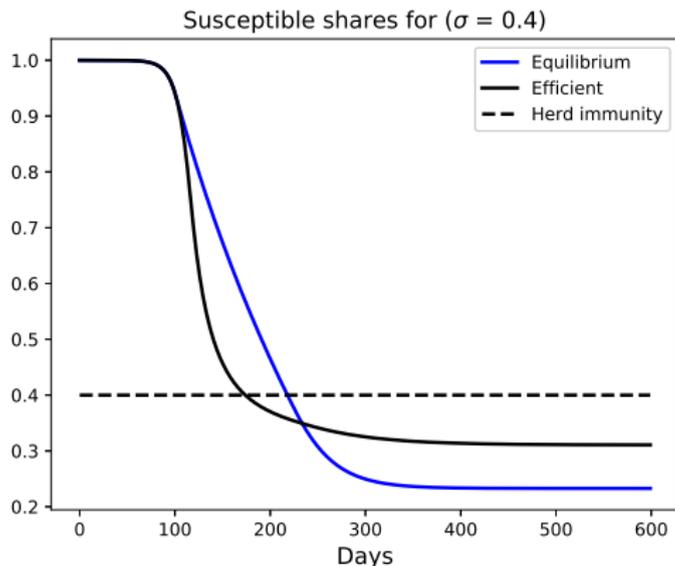
Recommended action on equilibrium path without vaccine



Welfare cost and death toll

σ	Welfare loss (%)			Death toll (per 100,000)		
	Myopic	PBE	SPP	Myopic	PBE	SPP
0.1	2.85	2.84	2.59	242	209	185
0.2	2.85	2.83	2.59	242	209	185
0.4	2.84	2.81	2.59	242	207	186
0.7	2.83	2.74	2.57	242	205	186
1.0	2.84	2.63	2.53	242	202	183

Efficient and equilibrium susceptible shares



Conclusion

- ▶ Theoretically studied equilibrium model of epidemics with imperfect testing and enforcement
- ▶ If a planner can intervene only temporarily, then some lockdown is optimal, though this incentive vanishes as testing becomes perfect
- ▶ Numerical findings:
 - ▶ Endogenous social distancing by agents mitigates epidemic, but welfare gain modest ($\sim 10\%$ reduction in welfare costs)
 - ▶ Additional welfare gain from lockdown policy small ($< 10\%$)
 - ▶ Quarantine effective, even with imperfect testing
 - ▶ When vaccine not expected soon, optimal policy is to encourage people to go out initially and only discourage around herd immunity level

Continuity of fixed point

Lemma

Let (X, d) be a complete metric space and Θ be a topological space. Endow $X \times \Theta$ with the product topology. Suppose $T : X \times \Theta \rightarrow X$ is continuous and there exists $\beta \in [0, 1)$ such that

$$d(T(x, \theta), T(y, \theta)) \leq \beta d(x, y)$$

for all $x, y \in X$ and all $\theta \in \Theta$. Then

1. for each $\theta \in \Theta$, there exists a unique $x^*(\theta) \in X$ such that $T(x^*(\theta), \theta) = x^*(\theta)$, and
2. $x^* : \Theta \rightarrow X$ is continuous.

▶ Go back

Lemma

Let $I \subset \mathbb{R}$ be an interval, $f : I \rightarrow \mathbb{R}$ continuously differentiable with $f' \neq 0$ on I , and $g : I \rightarrow \mathbb{R}$ continuous. If $f(a) = g(b)$ for some $a, b \in I$, then

$$|a - b| \leq \frac{|f(b) - g(b)|}{\min_{x \in [a, b]} |f'(x)|}.$$

Proof:

- ▶ By mean value theorem, there exists $\theta \in (0, 1)$ such that $f(b) - f(a) = f'(c)(b - a)$, where $c = (1 - \theta)a + \theta b$
- ▶ Since $f(a) = g(b)$, we obtain $|f(b) - g(b)| = |f(b) - f(a)| = |f'(c)(b - a)| \geq m |a - b|$, where $m = \min_{x \in [a, b]} |f'(x)| > 0$
- ▶ Dividing both sides by $m > 0$, we obtain the desired result

▶ Go back

