

Capital and Labor Income Pareto Exponents in the United States, 1916–2019

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Pareto tail

- Random variable $X > 0$ has Pareto upper tail if $P(X > x) \sim x^{-\alpha}$ for large x , where α : Pareto exponent
- Discovered by Pareto (1896) for income, but holds for other variables:
 - city size (Gabaix, 1999), $\alpha \sim 1$,
 - firm size (Axtell, 2001), $\alpha \sim 1$,
 - COVID cases (Beare and Toda, 2020), $\alpha \sim 1$ [▶ Picture](#)
 - household wealth (Klass et al., 2006; Vermeulen, 2018), $\alpha \sim 1.5$
 - household consumption (Toda and Walsh, 2015), $\alpha \sim 4$,
 - total income (Feenberg and Poterba, 1993; Atkinson and Piketty, 2010), $\alpha \sim 1.5-3$,
 - capital income (de Vries and Toda, 2021), $\alpha \sim 1.5$, etc.

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 - $g \in [0, 1]$: marginal utility weight on top earners,
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 - $g \in [0, 1]$: marginal utility weight on top earners,
 - e : elasticity of top income w.r.t. tax rate
- Calibration of macroeconomic models
 - average wealth of agents above some threshold is $\frac{\alpha}{\alpha-1}$ times threshold
 - hence wealthy agents have substantial impact on aggregate quantities
 - (see Beare and Toda (2022) for determining α in economic models and Gouin-Bonenfant and Toda (2022) for numerically solving models)

Accurately estimating income α is challenging

- **Limitation in data availability**
 - micro survey data (CPS, SCF, etc.) have small sample size $n = 10^3 \sim 10^4$
 - survey data suffer from low or inaccurate response
 - micro administrative data hard to access (IRS Public Use File available only for 2009–2014 with \$10,000 per year, noise added to data to protect confidentiality)

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- **Limitation in applicability of statistical methods**
 - common methods (Hill, 1975; Gabaix and Ibragimov, 2011) assume availability of micro data
 - maximum likelihood can be applied to grouped data if income thresholds observable, but IRS data provides income thresholds only for total income
- Hence existing estimates (i) rarely distinguish capital/labor income, (ii) are likely inaccurate, or (iii) are non-systematic

What we do

- Estimate capital and labor income Pareto exponents in U.S., 1916–2019, using best data and best estimation method
 - distinguishing capital/labor matters because taxed differently
- We use tabulated summaries from IRS *Statistics of Income*
 - administrative data from tax returns (likely accurate)
 - publicly available for 1916–2019
 - large sample size: $n = 10^6 \sim 10^8$
- We apply minimum distance method of Toda and Wang (2021) based on extreme value theory
 - can be applied to grouped data
 - no need to observe income thresholds
 - suffices to observe group averages

What we find

- Based on α , sample period can be divided into three sub periods, pre-1940, 1940–1985, and post-1985
- Post-1985, capital $\alpha \approx 1.2$, labor $\alpha \approx 2.0$
- α lower than existing estimates, hence higher top tail inequality (likely due to underreporting in survey)
- No systematic trend post-1985, so rise in income inequality measured by top income shares (Piketty and Saez, 2003) is inequality between rich and poor, not among rich

General framework

- Income $\{Y_i\}_{i=1}^n$, unobserved by researcher
- Top order statistics $Y_{(1)} \geq Y_{(2)} \geq \dots \geq Y_{(n)}$
- Partial sums of order statistics $S_m := \sum_{i=1}^m Y_{(i)}$
- Observables are $\{n_k, S_{n_k}\}_{k=1}^K$, where K is number of income groups and $n_1 < n_2 < \dots < n_K \leq n$

Example: U.S. 2019 tax returns data

| Income group | | Adjusted gross income (AGI) | | Salaries and wages | |
|--------------------|---------------|-----------------------------|----------------|--------------------|---------------|
| <i>k</i> | AGI threshold | # returns | Total income | # returns | Total income |
| 18 | \$1 | 9,866,880 | 24,439,988 | 6,672,531 | 23,927,191 |
| 17 | \$5,000 | 9,925,940 | 74,584,857 | 7,622,306 | 58,927,624 |
| 16 | \$10,000 | 11,087,737 | 138,230,399 | 8,277,447 | 100,631,554 |
| 15 | \$15,000 | 10,039,446 | 175,255,963 | 7,931,946 | 134,897,400 |
| 14 | \$20,000 | 9,493,968 | 213,660,160 | 7,855,283 | 173,142,941 |
| 13 | \$25,000 | 9,289,939 | 254,877,708 | 7,943,835 | 212,428,275 |
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| 11 | \$40,000 | 12,503,041 | 560,258,808 | 10,931,707 | 465,547,848 |
| 10 | \$50,000 | 22,238,948 | 1,366,892,948 | 18,976,338 | 1,071,062,478 |
| 9 | \$75,000 | 14,118,568 | 1,222,947,425 | 12,033,727 | 921,390,540 |
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| 7 | \$200,000 | 7,297,883 | 2,090,808,696 | 6,414,121 | 1,429,162,189 |
| 6 | \$500,000 | 1,162,371 | 781,920,814 | 1,010,488 | 449,489,139 |
| 5 | \$1,000,000 | 254,197 | 305,561,848 | 214,955 | 141,101,999 |
| 4 | \$1,500,000 | 103,075 | 176,961,208 | 85,285 | 72,754,006 |
| 3 | \$2,000,000 | 143,514 | 425,088,995 | 117,168 | 145,270,762 |
| 2 | \$5,000,000 | 34,738 | 237,781,553 | 28,162 | 66,367,353 |
| 1 | \$10,000,000 | 20,876 | 590,230,011 | 16,866 | 102,518,828 |
| All returns, total | | 157,796,807 | 11,966,873,976 | 129,775,754 | 8,273,071,046 |

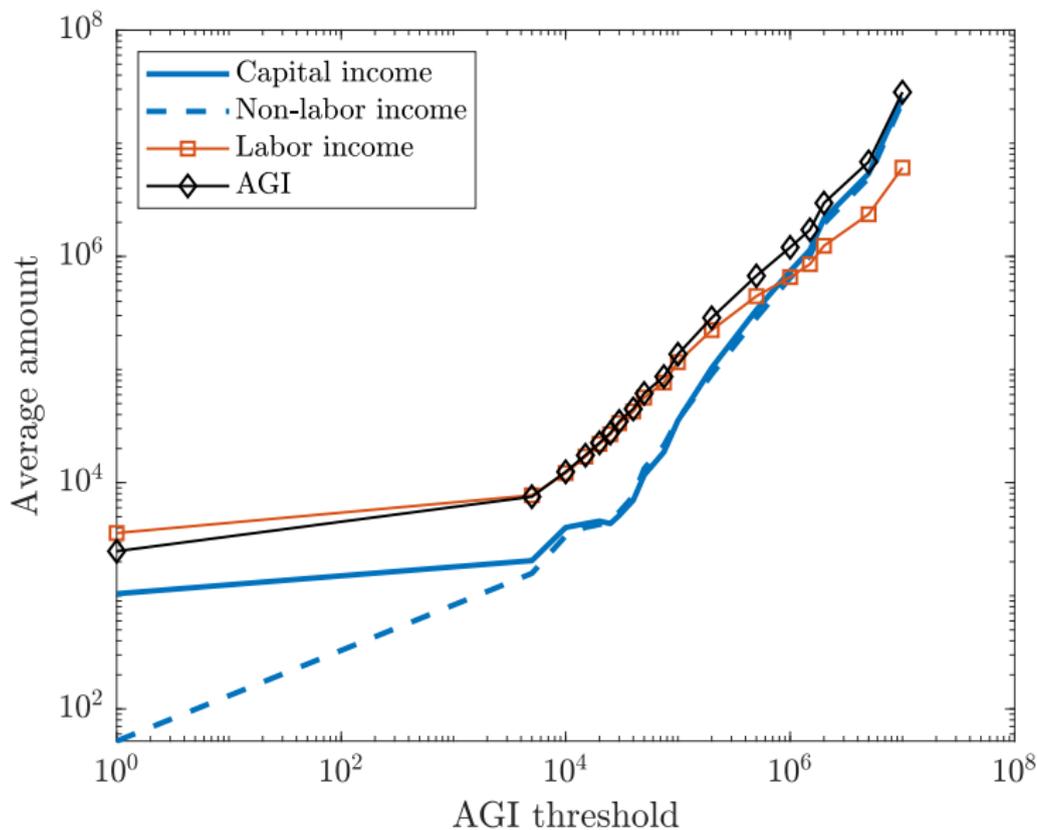
Details on data

- Primary data is *Statistics of Income (SOI) Individual Tax Returns Publication 1304* from IRS
 - Excel spreadsheets available since 1993 (Table 1.4 at <https://www.irs.gov/statistics/soi-tax-stats-individual-income-tax-returns-complete-report-publication-1304-basic-tables-2019>)
 - Before 1993, only PDFs of scanned copies of SOI are available (<https://www.irs.gov/statistics/soi-tax-stats-archive>)
- Before 1993, manually input adjusted gross income (AGI), AGI thresholds, salaries and wages, and number of returns into spreadsheets
- Human errors inevitable (for each year, we typed 10-digit numbers 100 times); checked accuracy by comparing column sums of spreadsheet to sums reported in SOI tables

Definitions of incomes

- We define
 - Total income := “adjusted gross income (AGI)”
 - Labor income := “salaries and wages”
 - Capital income := non-labor income = $AGI - \text{labor income}$
- This definition of capital income is broad and includes clearly non-capital income such as “state income tax refunds”, “alimony received”, “unemployment compensation”
- Hence also consider adding up capital income components such as “taxable interest”, “tax-exempt interest”, “ordinary dividends”, “qualified dividends”, “business or profession”, “capital gain distributions reported on Form 1040”, “sales of capital assets reported on Form 1040, Schedule D”, “sales of property other than capital assets”, “taxable Individual Retirement Arrangement (IRA) distributions”, “pensions and annuities”, “total rent and royalty”, “partnership and S corporation”, “estate and trust”

Capital income \approx non-labor income for AGI $>$ 25k

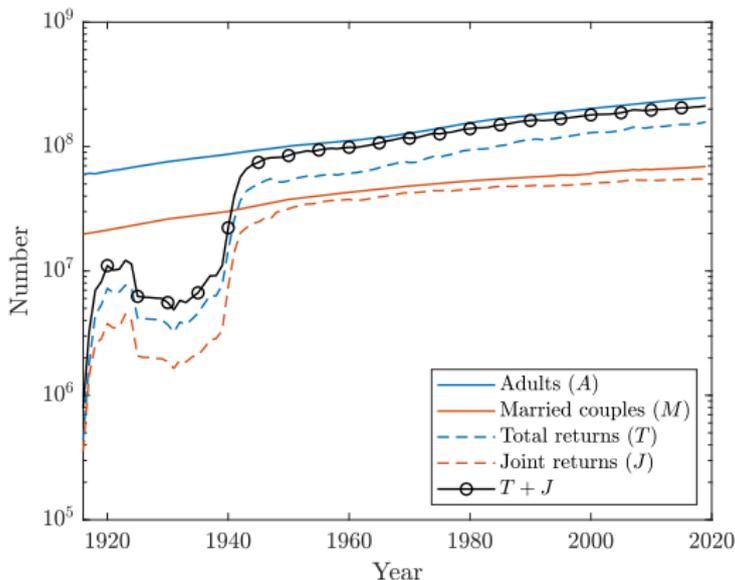


Sample size

- Our unit of analysis is tax unit
 - individuals or married couples filing jointly, with dependents if any
- We only observe tax filers
 - non-filer could have income below filing requirement or work in informal sectors using cash and evade taxes
- Sample size (number of potential tax units) necessary for estimation (definition of top fractiles)
- To estimate sample size, we collect data on
 - number of total returns (T),
 - number of joint returns (J),
 - number of adults (A),
 - number of married couples (M)

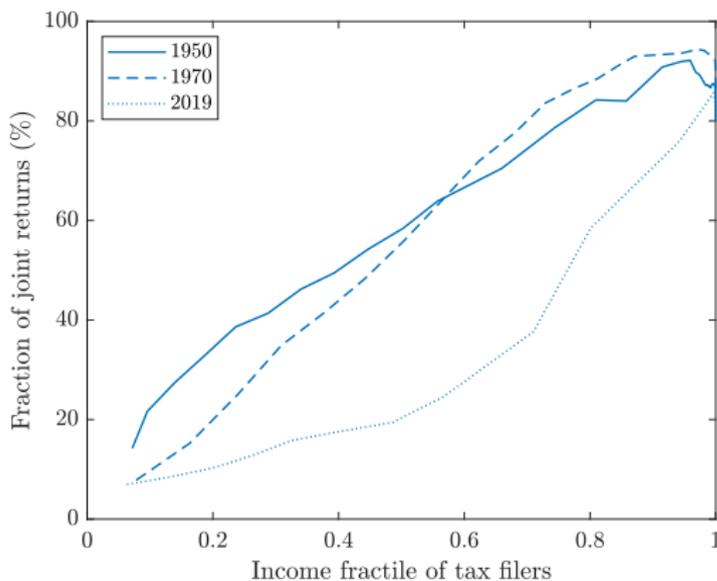
Number of adults and tax returns

- If $\{\text{Adults}\} = \{\text{Tax filers}\}$, then $A = T + J$
 - Post-1950, $(T + J)/A \approx 0.9$, so missing about 10% of adults
 - Pre-1945, missing 90-99% of adults due to high exemptions (Tax Reform Act of 1942)



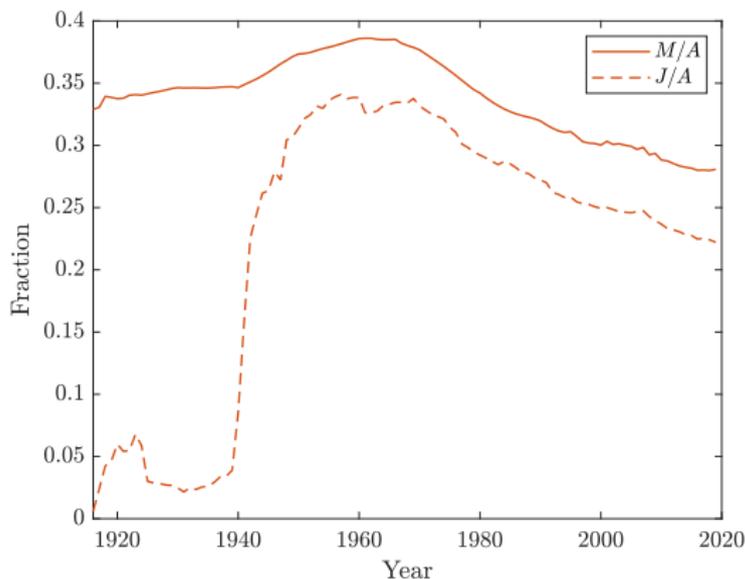
Income fractiles and joint returns

- Low (high) income earners tend to file separately (jointly)
- Hence can estimate sample size as $n = A - J$
- Still need to estimate J for pre-1950



Married couples and joint returns

- Post-1950, married/adults (M/A) and joint/adults (J/A) have common trends
- Regress $\log(J/A)$ on $\log(M/A)$ ($R^2 = 0.989$) post-1950 and use OLS estimates to construct \hat{J} pre-1950



Estimation

- We now have data on $\{(n_k, S_{n_k})\}_{k=1}^K$ and n , where $n_1 < n_2 < \dots < n_K \leq n$
- We apply the minimum distance method of Toda and Wang (2021) (TW) to estimate income Pareto exponents
- Here are basic idea of TW method
 - Letting $J : [0, 1] \rightarrow \mathbb{R}$ a bounded and almost everywhere continuous function, the asymptotic behavior of weighted sums of order statistics

$$\frac{1}{n} \sum_{i=1}^n J\left(\frac{i}{n+1}\right) Y_{(n-i+1)}$$

is known (Stigler, 1974)

- Let $p_k = n_k/n$ be top fractile

Estimation

- Basic idea (continued)
 - Letting $J(x) = \mathbb{1}(1 - p_{n_{k+1}} < x \leq 1 - p_{n_k})$, we have

$$\frac{1}{n} \sum_{i=1}^n J\left(\frac{i}{n+1}\right) Y_{(n-i+1)} = \frac{S_{n_{k+1}} - S_{n_k}}{n}$$

- Hence if we consider self-normalized quantity

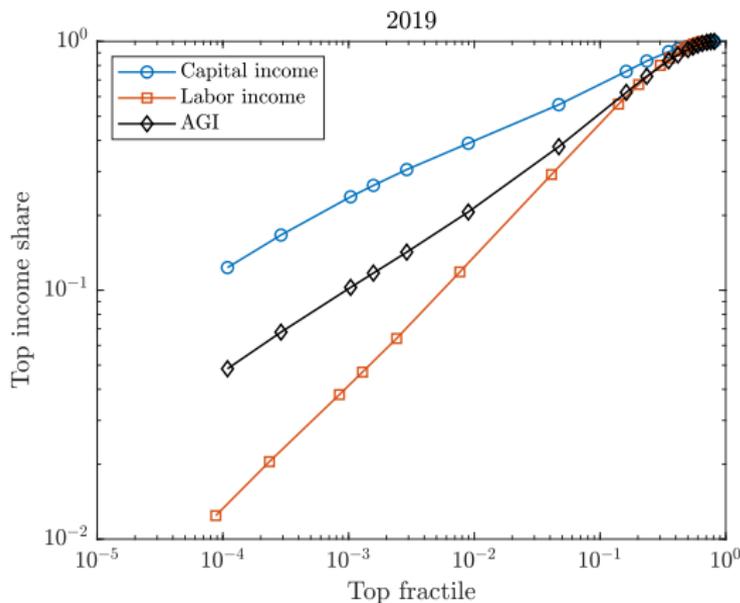
$$s := \left(\frac{S_{n_2} - S_{n_1}}{S_{n_{L+1}} - S_{n_L}}, \dots, \frac{S_{n_L} - S_{n_{L-1}}}{S_{n_{L+1}} - S_{n_L}} \right),$$

asymptotic behavior depends only on Pareto exponent α if $\{Y_i\}$ have Pareto upper tail and $n_{L+1} \ll n$

- Can estimate α by minimizing quadratic distance of s from theoretical value implied by Pareto distribution

Is Pareto tail reasonable?

- If income has Pareto upper tail with exponent α , top p fractile income share is $S(p) \propto p^{1-1/\alpha}$
- Hence top fractiles and shares should be linear in log-log scale

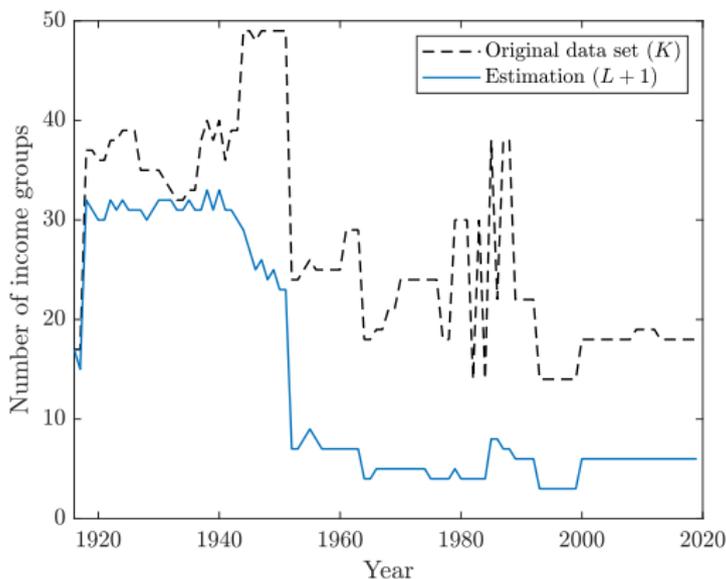


Assumptions

- For estimation, we need partial sums of order statistics
- In IRS data, income groups are defined by order of AGI
- We assume AGI and capital income are ordered in the same way *across* income groups (e.g., tax filers in group k have higher capital income than those in group $k + 1$ for $k = 1, \dots, L$)
 - Reasonable for capital income if L not too large because AGI \approx capital income for top earners ▶ average income
 - Unreasonable for labor income because labor income \ll AGI for top earners

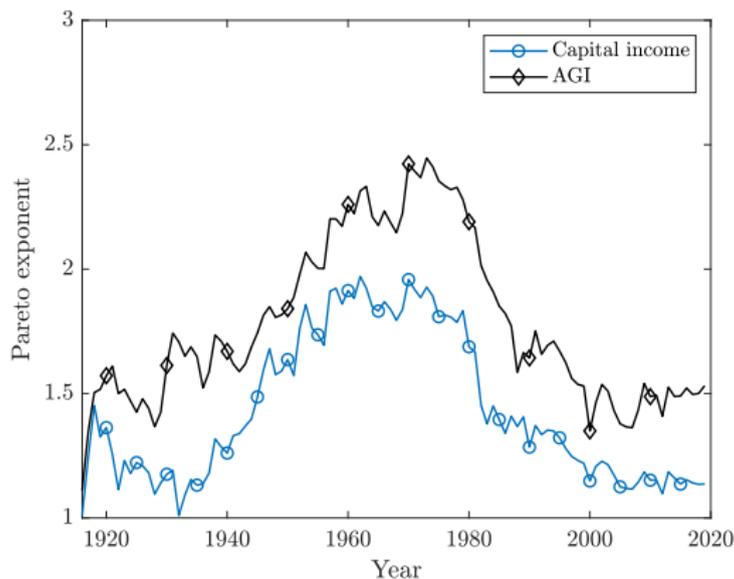
Choice of income groups

- Literature typically uses top 5% observation for estimation
- We choose largest L such that $n_{L+1}/n \leq 0.01$ (top 1%) to be conservative, given large sample size (and need $L + 1 \geq 3$)
- Results are robust to different cutoffs [▶ robustness](#)



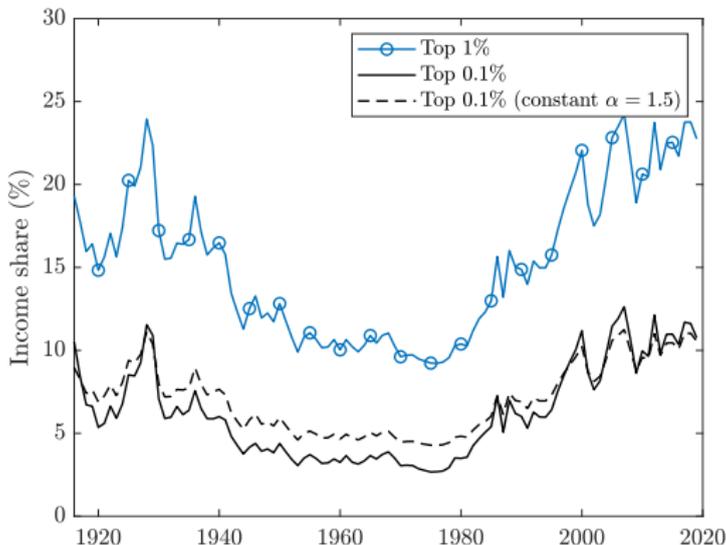
AGI and capital income Pareto exponents

- Capital $\alpha \approx 1.2$ pre-1940 and post-1985, inverse U-shape in 1940–1985, AGI α similar pattern
- Standard error omitted because order of magnitude $(10^8 \times 0.01)^{-1/2} = 10^{-3}$



No rise in top tail inequality post-1985

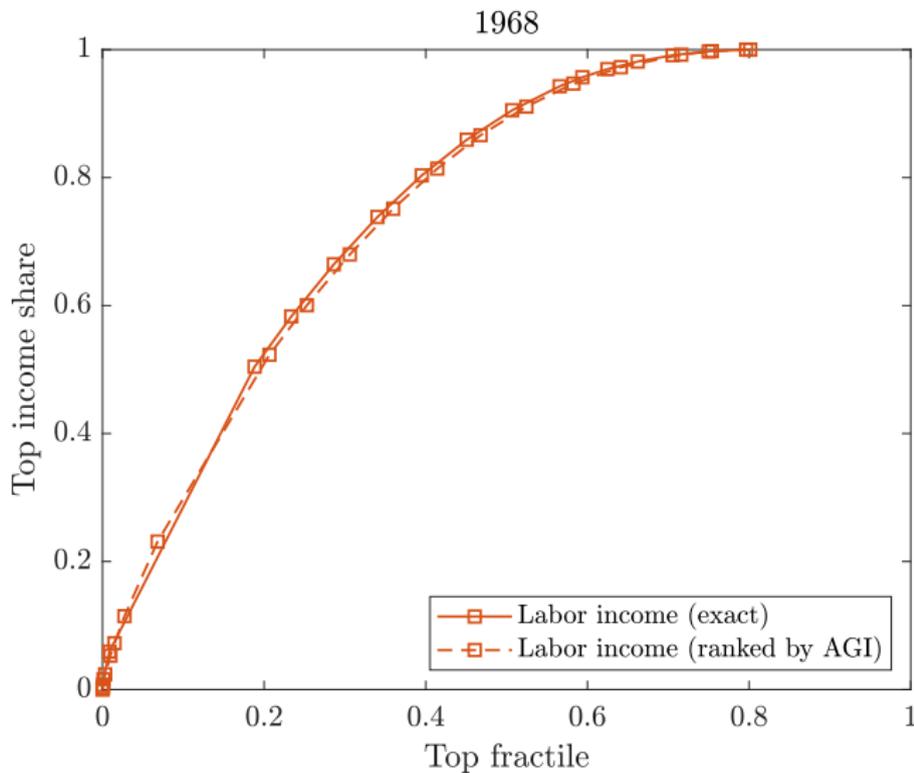
- Top income shares have risen post-1985
- If income Pareto, then $S(p) = (p/q)^{1-1/\alpha} S(q)$
- $S(p)$ constructed from $(p, q) = (0.001, 0.01)$ and $\alpha = 1.5$ is similar to actual $S(p)$ post-1985, confirming stable top tail inequality



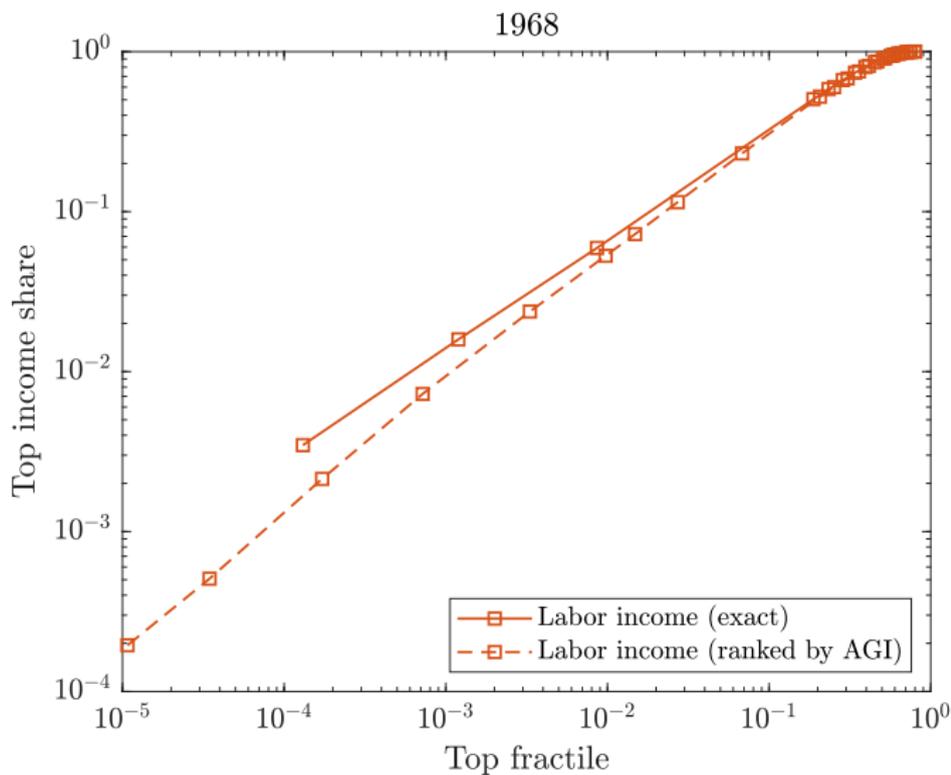
Labor income Pareto exponents

- Several issues when estimating labor income exponents
 - Not necessarily reasonable to assume same ordering of AGI and labor income across income groups
 - Size distribution of labor income available only for subset of 1927–1978
- For a particular year (1968) the joint distribution of AGI and labor income is available, compare top labor income shares

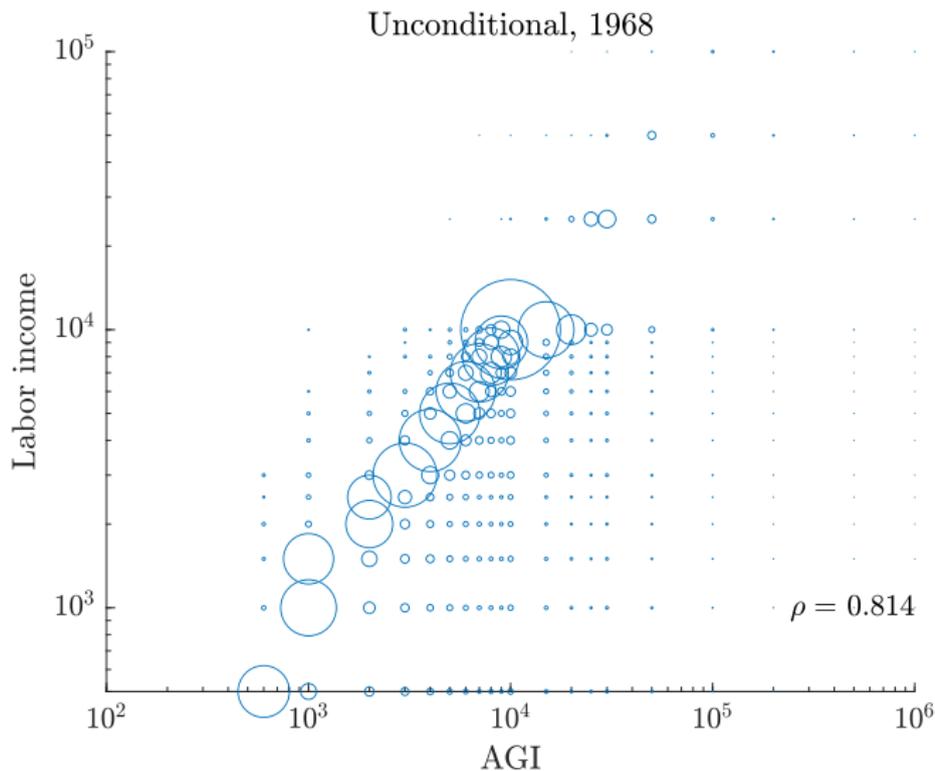
Top labor income shares, 1968



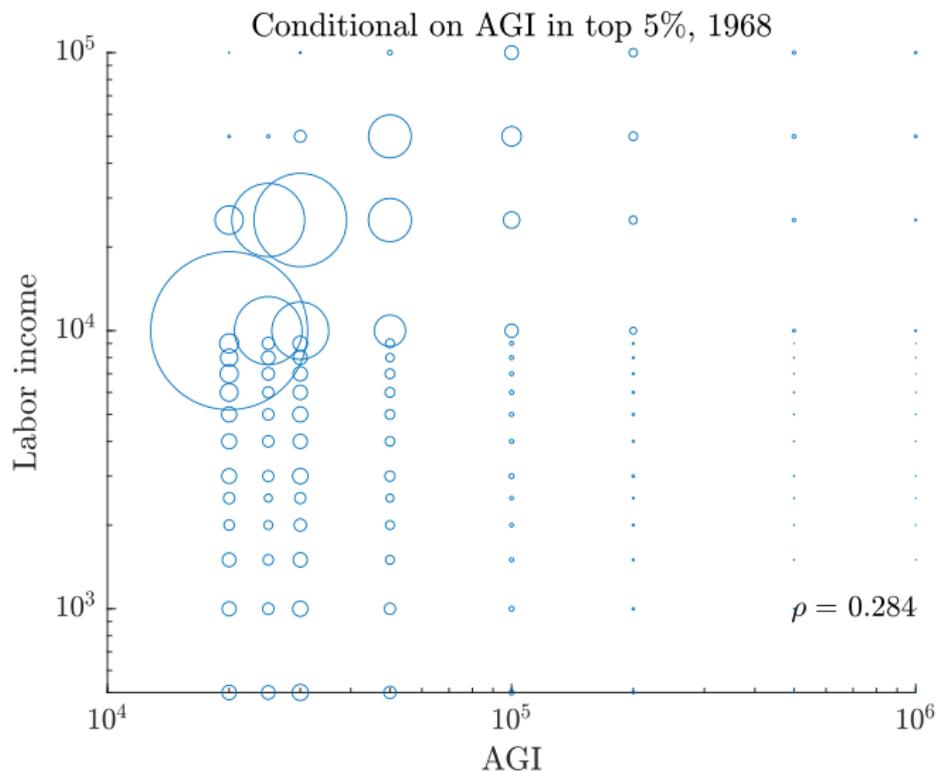
Top labor income shares, 1968



Joint distribution of AGI and labor income

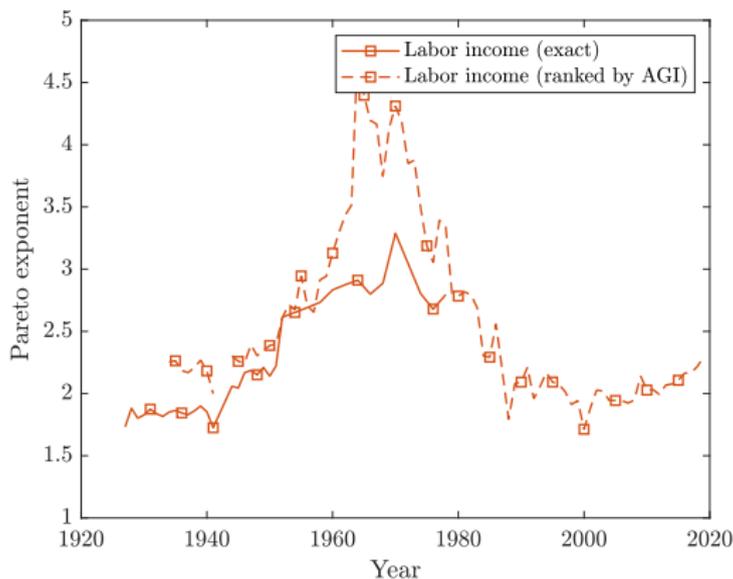


Joint distribution of AGI and labor income



Labor income Pareto exponents

- These figures suggest that AGI and labor income are highly correlated, but excluding top incomes
- We simply report two numbers using labor income ranked exactly (1927–1978) and ranked by AGI (1934–2019)



Comparison to existing estimates

- We compare to existing estimates,
 - maximum likelihood (only AGI due to applicability of estimation method),
 - Feenberg and Poterba (1993) (only AGI due to applicability of estimation method),
 - Atkinson and Piketty (2010) (only AGI due to applicability of estimation method),
 - de Vries and Toda (2021) (capital and labor income exponents)

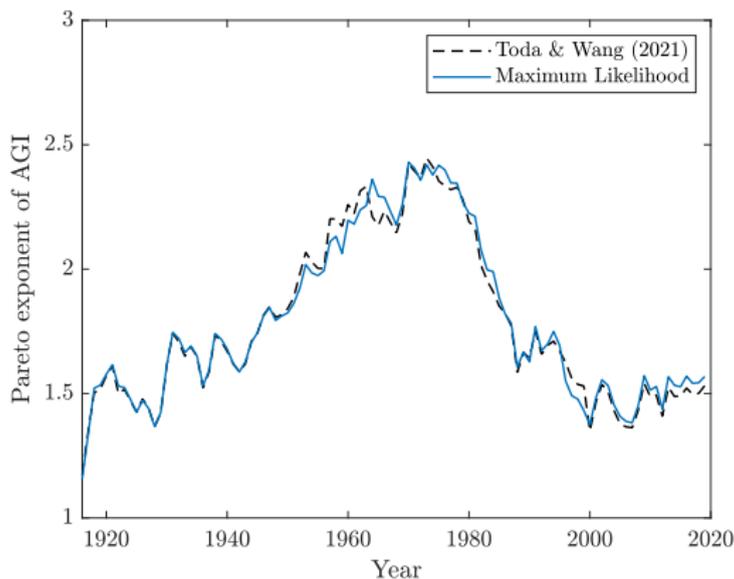
Maximum likelihood

- For AGI, we observe income thresholds

$$\infty = t_0 > t_1 > \dots > t_K > 0$$

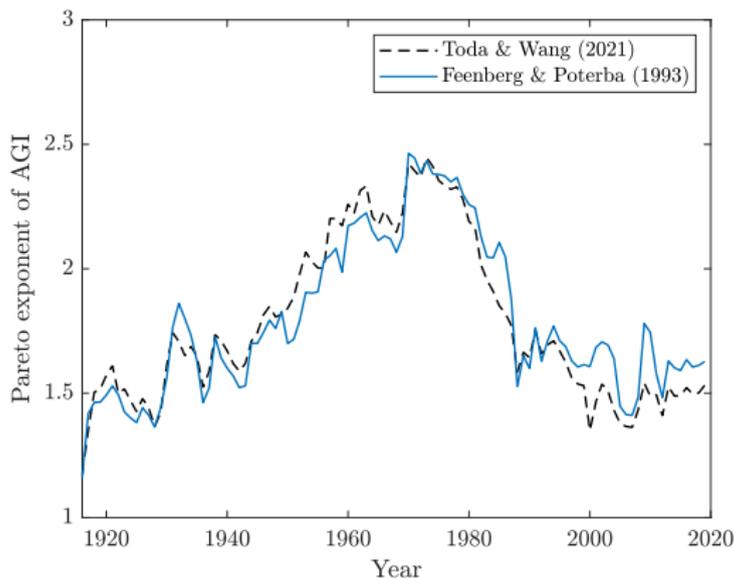
- Can apply ML using conditional tail probability

$$P(Y \geq y \mid Y \geq t_L) = (y/t_L)^{-\alpha}$$



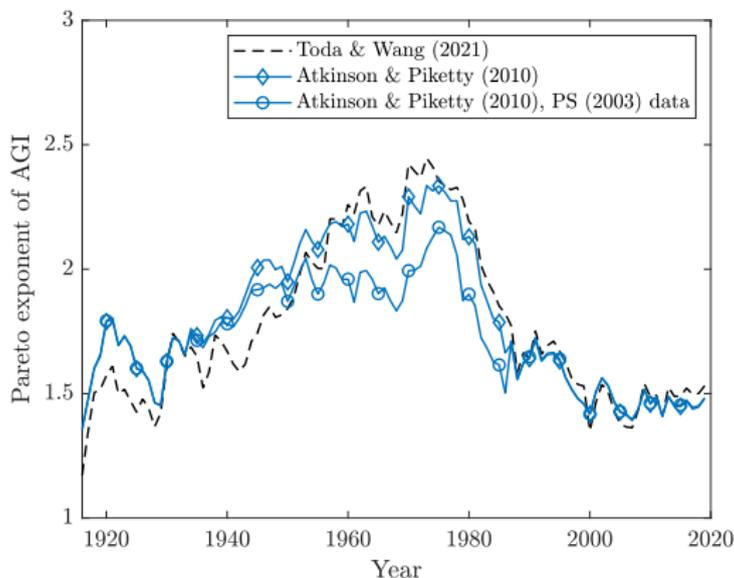
Feenberg and Poterba (1993)

- Feenberg and Poterba (1993) find two income thresholds $y_1 < y_2$ that bracket the top 0.5%
- Estimate $\hat{\alpha} = \log[(1 - F(y_1))/(1 - F(y_2))]/\log(y_2/y_1)$ using Pareto CDF



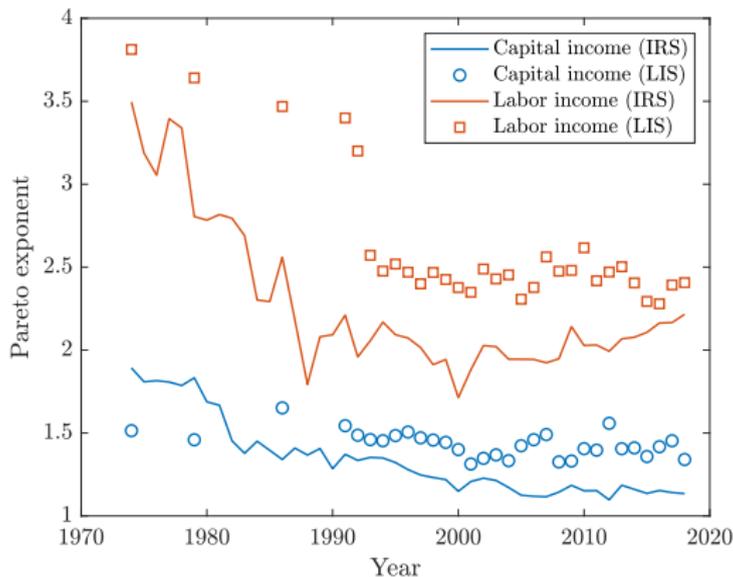
Atkinson and Piketty (2010)

- If income Pareto, top p fractile income share is $S(p) \propto p^{1-1/\alpha}$
- Estimate $\hat{\alpha} = \left(1 - \frac{\log[S(q)/S(p)]}{\log(q/p)}\right)^{-1}$ using top income shares of Piketty and Saez (2003) ▶ difference?



de Vries and Toda (2021)

- de Vries and Toda (2021) apply Hill (1975) estimator to micro data (*Luxembourg Income Study*, which is CPS for U.S.)
- Estimates from survey data biased upwards, suggesting low response/underreporting by rich



Concluding remarks

- First systematic estimates of capital and labor income Pareto exponents in U.S., 1916–2019
- Post-1985, exponents stable at capital $\alpha \approx 1.2$, labor $\alpha \approx 2.0$
- α lower than existing estimates (higher top tail inequality)
- No systematic trend post-1985, so rise in income inequality is inequality between rich and poor, not among rich

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Statistics of Income 2019

| A | | D | | E | | F | | G | |
|-------------------------------|-------------------------------|--------------------|-----------------------|--------------------|----------------------|---|--|---|--|
| | | Total income | | Salaries and wages | | | | | |
| Size of adjusted gross income | | Number of returns | Amount | Number of returns | Amount | | | | |
| | | (3) | (4) | (5) | (6) | | | | |
| 9 | All returns, total | 157,197,473 | 12,111,799,488 | 129,775,754 | 8,273,071,046 | | | | |
| 10 | No adjusted gross income | 1,528,166 | -234,790,580 | 569,047 | 23,421,857 | | | | |
| 11 | \$1 under \$5,000 | 9,866,880 | 25,527,613 | 6,672,531 | 23,927,191 | | | | |
| 12 | \$5,000 under \$10,000 | 9,925,940 | 76,354,338 | 7,622,306 | 58,927,624 | | | | |
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| 25 | \$1,500,000 under \$2,000,000 | 103,075 | 179,090,416 | 85,285 | 72,754,006 | | | | |

Statistics of Income 1919

https://www.irs.gov/pub/irs-soi/1919soirepar.pdf

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Automatic Zoom

TABLE 2.—PERSONAL RETURNS—DISTRIBUTION BY INCOME CLASSES, FOR THE UNITED STATES; showing for each class of income, the number of returns, net income, personal exemption, dividends, tax paid, and percentages.

[Income returned for the calendar year ended Dec. 31, 1919.]

| Income class. | Number of returns. | Net income. | Exemptions from normal tax. | | | Normal tax. | Surtax. | Total tax. | Average amount of tax per individual. | Average rate of tax per cent. |
|---------------------------------------|--------------------|---------------|-----------------------------|--------------|--|--------------|--------------|-------------|---------------------------------------|-------------------------------|
| | | | Personal exemption. | Dividends. | Interest on Government obligations. ¹ | | | | | |
| \$1,000 to \$2,000 ² | 536,046 | \$829,331,321 | \$1,097,674,000 | \$30,009,958 | | | | | | |
| \$2,000 to \$3,000..... | 1,388,826 | 1,969,781,800 | 1,352,866,700 | 13,579,367 | \$6,492 | \$24,096,200 | \$24,096,200 | \$17.78 | 1.23 | |
| \$3,000 to \$5,000..... | 329,293 | 1,297,458,228 | 1,290,453,300 | 57,771,942 | | | | | | |
| \$5,000 to \$7,000..... | 1,049,472 | 2,599,427,039 | 1,569,829,100 | 35,956,156 | 89,797 | 28,287,861 | 28,287,861 | 27.16 | 1.00 | |
| \$7,000 to \$10,000..... | 27,305 | 80,743,612 | 61,673,500 | 38,720,139 | 6,239 | | | | | |
| \$10,000 to \$14,000..... | 715,029 | 2,442,318,394 | 1,476,225,400 | 52,715,984 | 200,583 | 37,688,780 | 37,688,780 | 52.67 | 1.33 | |
| \$14,000 to \$18,000..... | 8,959 | 39,347,529 | 16,574,100 | 33,191,326 | | | | | | |
| \$18,000 to \$25,000..... | 429,195 | 1,925,596,995 | 857,842,000 | 72,522,402 | 456,465 | 38,256,067 | 38,256,067 | 88.13 | 1.96 | |
| \$25,000 to \$3,000..... | 167,065 | 913,242,227 | 247,915,700 | 69,040,834 | 3,928,550 | 20,030,359 | \$80,928 | 20,827,885 | 128.77 | 2.28 |
| \$3,000 to \$7,000..... | 169,674 | 704,235,482 | 228,877,000 | 71,644,145 | 3,067,587 | 17,210,561 | 2,006,798 | 19,217,359 | 178.22 | 2.73 |
| \$7,000 to \$10,000..... | 75,719 | 549,065,475 | 150,724,600 | 65,148,884 | 2,870,040 | 15,415,438 | 2,777,062 | 18,292,100 | 248.15 | 3.33 |
| \$10,000 to \$15,000..... | 50,486 | 427,541,591 | 102,796,000 | 60,051,704 | 2,445,223 | 13,088,701 | 3,171,449 | 16,860,150 | 333.96 | 3.94 |
| \$15,000 to \$18,000..... | 37,967 | 309,294,668 | 76,802,600 | 56,959,770 | 2,088,305 | 12,747,896 | 2,381,610 | 15,329,416 | 430.09 | 4.53 |
| \$18,000 to \$11,000..... | 28,499 | 298,741,205 | 57,115,000 | 55,548,052 | 1,895,861 | 11,115,245 | 3,045,637 | 14,888,882 | 519.63 | 4.96 |
| \$11,000 to \$13,000..... | 22,541 | 262,547,302 | 45,085,400 | 48,218,135 | 1,769,071 | 10,344,700 | 3,857,400 | 14,202,000 | 627.78 | 5.41 |
| \$13,000 to \$15,000..... | 18,423 | 230,074,304 | 36,701,800 | 44,080,679 | 1,211,800 | 9,012,024 | 3,940,208 | 13,452,232 | 730.19 | 5.83 |
| \$15,000 to \$14,000..... | 15,248 | 205,889,632 | 30,323,200 | 40,337,100 | 1,290,592 | 8,598,608 | 4,017,757 | 12,716,366 | 833.97 | 6.18 |
| \$14,000 to \$15,000..... | 12,541 | 180,215,507 | 25,310,600 | 39,885,022 | 1,169,745 | 7,084,373 | 4,100,174 | 12,084,547 | 941.06 | 6.48 |
| \$15,000 to \$20,000..... | 42,028 | 724,378,155 | 82,021,000 | 102,856,875 | 4,896,637 | 32,280,924 | 20,918,502 | 35,205,426 | 1,263.96 | 7.34 |
| \$20,000 to \$25,000..... | 22,605 | 504,458,801 | 41,590,200 | 132,167,487 | 3,649,963 | 20,483,480 | 29,879,536 | 44,365,016 | 1,992.53 | 8.79 |
| \$25,000 to \$30,000..... | 13,769 | 376,457,979 | 26,438,800 | 103,083,805 | 2,723,645 | 17,673,337 | 20,271,767 | 37,947,304 | 2,756.00 | 10.08 |
| \$30,000 to \$40,000..... | 15,410 | 520,754,145 | 29,000,400 | 108,022,816 | 4,137,076 | 25,029,230 | 36,061,884 | 63,175,124 | 4,099.02 | 11.90 |
| \$40,000 to \$50,000..... | 8,298 | 379,152,311 | 15,626,600 | 121,827,016 | 2,542,469 | 17,948,138 | 25,876,777 | 45,325,915 | 6,486.37 | 14.54 |
| \$50,000 to \$60,000..... | 5,213 | 284,766,454 | 9,669,400 | 99,625,332 | 1,968,291 | 13,764,825 | 34,864,564 | 48,629,371 | 9,328.48 | 17.08 |
| \$60,000 to \$70,000..... | 3,196 | 206,616,321 | 5,802,000 | 74,315,645 | 1,762,479 | 9,818,475 | 20,326,310 | 49,144,785 | 12,500.05 | 19.44 |
| \$70,000 to \$80,000..... | 2,227 | 167,032,648 | 4,271,400 | 49,453,629 | 1,302,929 | 7,864,723 | 28,444,808 | 36,309,600 | 16,700.00 | 21.86 |
| \$80,000 to \$90,000..... | 1,561 | 132,629,947 | 2,847,800 | 48,816,222 | 1,125,061 | 6,433,707 | 26,120,745 | 32,660,452 | 20,838.71 | 24.55 |
| \$90,000 to \$100,000..... | 1,113 | 105,520,839 | 2,012,200 | 38,384,249 | 966,971 | 5,098,466 | 25,414,664 | 28,513,400 | 25,616.00 | 27.02 |
| \$100,000 to \$150,000..... | 2,983 | 368,322,663 | 5,736,200 | 146,154,738 | 3,757,523 | 17,801,833 | 101,303,459 | 118,705,303 | 39,796.00 | 34.12 |
| \$150,000 to \$200,000..... | 1,092 | 187,816,010 | 1,903,200 | 82,854,402 | 2,264,238 | 8,861,562 | 68,261,248 | 77,127,790 | 70,487.87 | 40.98 |
| \$200,000 to \$250,000..... | 522 | 115,428,091 | 985,000 | 46,336,471 | 1,874,064 | 5,682,396 | 47,071,793 | 52,754,132 | 101,001.56 | 43.71 |
| \$250,000 to \$300,000..... | 250 | 67,844,435 | 494,400 | 29,531,000 | 864,696 | 2,253,383 | 30,114,879 | 33,368,467 | 132,473.57 | 69.14 |
| \$300,000 to \$400,000..... | 285 | 96,611,153 | 494,800 | 46,489,968 | 1,008,507 | 4,615,033 | 46,209,076 | 50,824,129 | 178,330.28 | 52.61 |

¹ Interest on Government obligations not wholly exempt from tax.

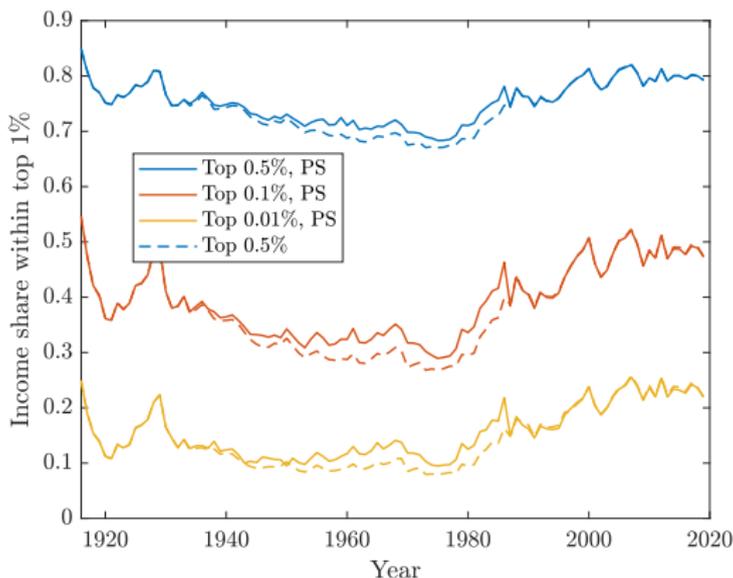
² Nontaxable. Personal exemption and dividends exceed net income.

STATISTICS OF INCOME.

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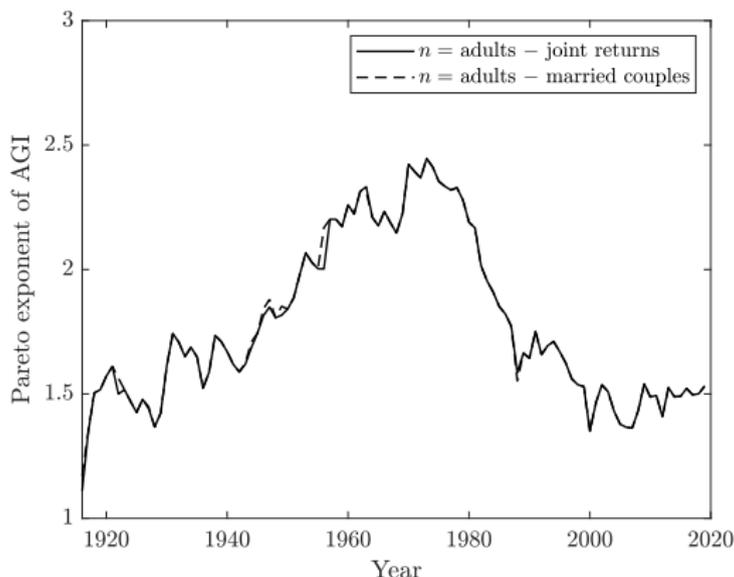
Top relative income shares

- If income Pareto, top p fractile income share is $S(p) \propto p^{1-1/\alpha}$
- Hence top relative income share $S(p)/S(q) = (p/q)^{1-1/\alpha}$ depends only on α and top relative fractile p/q



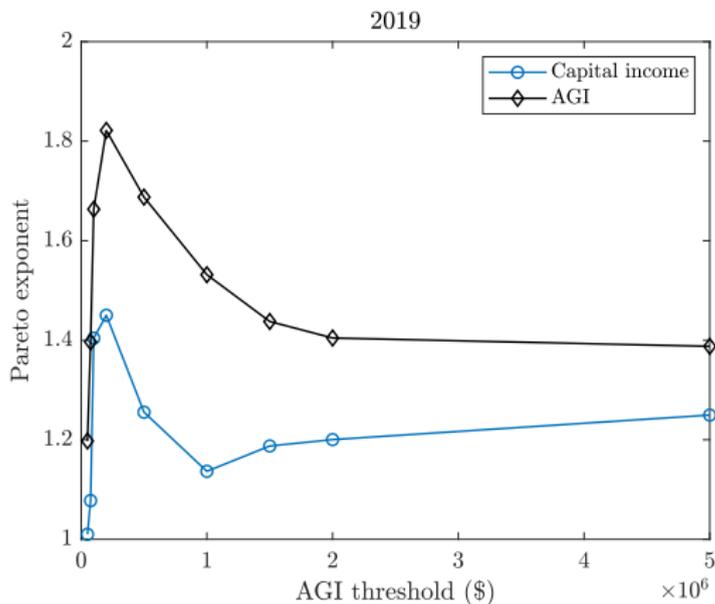
Robustness to sample size

- We use $n = A - J$ for sample size
- Using $n = A - M$ has no material impact



Robustness to choice of income groups

- We use top 1% for estimation



Size distribution of COVID cases across U.S. counties

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