Leverage, Endogenous Unbalanced Growth, and Asset Price Bubbles[1]

Tomohiro Hirano[1] Ryo Jinnai[2] Alexis Akira Toda[3]

[1]Royal Holloway, University of London

[2]Hitotsubashi University

[3]Emory University

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Asset price bubbles

- Casual inspection of modern economic history suggests presence of asset price bubbles
- Examples:
  - Japanese real estate bubble in late 1980s ▸ Image
  - U.S. dot-com bubble in late 1990s
  - U.S. housing bubble in mid 2000s
- Kindleberger (2000) documents 38 bubbly episodes in 1618–1998 period
- Rational bubble: asset price $(P) >$ fundamental value (V)
  - $V =$ present value of dividends (D)

Fundamental difficulties

- Dominant view in macro-finance: asset price reflects fundamentals ($P = V$)
- Fundamental difficulty in generating bubbles attached to dividend-paying assets
  - If D non-negligible relative to aggregate endowment, then $P = V$ (Santos and Woodford, 1997)
  - Bubble exists if and only if dividend yield D_t/P_t summable (Montrucchio, 2004)
  - Hence bubbles impossible in stationary models with $D > 0$
- Given fundamental difficulties, existing literature almost exclusively focus on $D = 0$ (pure bubble)
  - Many shortcomings including lack of realism, equilibrium indeterminacy, inability to connect to empirical literature

This paper

- Proposes macro-finance theory to think about bubbles attached to dividend-paying assets
- Keyword: unbalanced growth, unlike stationary world characterized by balanced growth
- Balanced or unbalanced growth endogenously determined by financial leverage:
  - Low leverage: weak financial accelerator, balanced growth, fundamental value
  - High leverage: strong financial accelerator, unbalanced growth, bubble
- Hence with financial (or technological) development and presence of fixed factor like land, bubbles become inevitable

Related literature

- Financial accelerator: Greenwald and Stiglitz (1993), Kiyotaki and Moore (1997), Lorenzoni (2008), Brunnermeier and Sannikov (2014) etc.
- Unbalanced growth: Baumol (1967), Matsuyama (1992), Buera and Kaboski (2012), etc.
- Rational bubbles: Samuelson (1958), Bewley (1980), Tirole (1985), Scheinkman and Weiss (1986), Kocherlakota (1992), Hirano and Toda (2024a,c), etc.
- Necessity of bubbles: Wilson (1981), Hirano and Toda (2024b)

Model overview

Key features of model:

- Infinite horizon: $t = 0, 1, \dots$
- Infinitely-lived heterogeneous agents
- Incomplete markets
- Leverage constraint
- AK model at individual level, neoclassical production with land at aggregate level

Agents

- Continuum of agents with mass 1 indexed by $i \in I = [0, 1]$
- Log utility (inessential; only for tractability)

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t$$

- Rational expectations

Productivity

- AK model at individual level: invest $i_t \to$ capital $k_{t+1} = z_t i_t$
- Productivity z_t drawn from cdf Φ, IID across agents and time
- Note: productivity known before investment

Assumption

$\Phi : [0, \infty) \to [0, 1)$ *is absolutely continuous with* $\int_0^\infty z \, \mathrm{d}\Phi(z) < \infty$.

Production

- Aggregate capital K, land X
- Neoclassical production function $f(K, X)$
  - CES example: $f(K, X) = A(\alpha K^{1-\rho} + (1-\alpha) X^{1-\rho})^{\frac{1}{1-\rho}}$
  - $1/\rho$ is elasticity of substitution
- Let $F(K, X) = f(K, X) + (1-\delta)K$, where δ: capital depreciation rate

Assumption

$F : \mathbb{R}^2_{++} \to \mathbb{R}_+$ is homogeneous of degree 1, concave, continuously differentiable with positive partial derivatives, and satisfies

$$\lim_{K \to \infty} \frac{F(K, 1)}{K} = \lim_{K \to \infty} F_K(K, 1) =: m > 0.$$

Land and REIT

- Land in unit supply
- In background, perfectly competitive financial intermediaries securitize land into real estate investment trust (REIT); hence land is just a risk-free asset (bond)
- Land rent at time t is $F_X(K_t, 1)$
- Hence gross risk-free rate from t to $t+1$ is

$$R_t := \frac{P_{t+1} + F_X(K_{t+1}, 1)}{P_t},$$

where P_t: land price

Budget constraint

- Let w_t be wealth of typical agent carried over from previous period
- Then agent decides allocation

$$\underbrace{c_t}_{\text{consumption}} + \underbrace{i_t}_{\text{investment}} + \underbrace{b_t}_{\text{bond}} = w_t$$

- Because capital is $k_{t+1} = z_t i_t$, time $t+1$ wealth is defined by

$$w_{t+1} := \underbrace{F_K(K_{t+1}, 1) z_t i_t}_{\text{income from capital}} + \underbrace{R_t b_t}_{\text{income from REIT}}.$$

Leverage constraint

- Agents are subject to leverage constraint

$$0 \leq \underbrace{i_t}_{\text{investment}} \leq \lambda(\underbrace{i_t + b_t}_{\text{equity}}),$$

  - $\lambda \geq 1$: exogenous leverage limit
  - $i_t + b_t = w_t - c_t$: "equity"

- Noting $1 - 1/\lambda \geq 0$, we obtain

$$\underbrace{-b_t}_{\text{borrowings}} \leq \underbrace{(1 - 1/\lambda)}_{\text{collateral ratio}} i_t,$$

so leverage constraint equivalent to borrowing constraint

Equilibrium

- Due to log utility, optimal consumption $c_t = (1 - \beta) w_t$
- Hence can aggregate individual behavior and rational expectations equilibrium reduces to aggregate dynamics of
  - R_t: gross risk-free rate
  - $\bar{z}_t$: productivity threshold for investment
  - W_t: aggregate wealth
  - K_t: aggregate capital
  - P_t: land price
- Below, we derive aggregate dynamics

Investment

- If produce capital, return on investment is $F_X(K_{t+1}, 1)z_t$
- If invest in risk-free asset, return is R_t
- Hence productivity threshold is

$$F_K(K_{t+1}, 1)z_t > R_t \iff z_t > \bar{z}_t := \frac{R_t}{F_K(K_{t+1}, 1)}$$

- Due to leverage constraint, optimal asset allocation is

$$(i_t, b_t) = \begin{cases} (0, \beta w_t) & \text{if } z_t \leq \bar{z}_t, \\ (\lambda \beta w_t, (1-\lambda)\beta w_t) & \text{if } z_t > \bar{z}_t. \end{cases}$$

Aggregate wealth

- Let $W_t := \int_I w_{it} \, \mathrm{d}i$ be aggregate wealth
- Recall individual wealth $w_t = F_X(K_t, 1)k_t + R_{t-1}b_{t-1}$
- Aggregating and noting bonds backed by land,

$$\begin{aligned} W_t &:= F_K(K_t, 1)K_t + R_{t-1}P_{t-1} \\ &= F_K(K_t, 1)K_t + P_t + F_X(K_t, 1) = F(K_t, 1) + P_t, \end{aligned}$$

where last line follows from

- homogeneity of F,
- definition of risk-free rate R_{t-1}

Aggregate capital and land price

- Aggregating individual capital $k_{t+1} = z_t i_t$, aggregate capital is

$$K_{t+1} = \beta\lambda W_t \int_{\bar{z}_t}^{\infty} z \, \mathrm{d}\Phi(z)$$

- Aggregating bond holdings b_t and noting bonds backed by land, land price is

$$P_t = \underbrace{\beta W_t \int_0^{\bar{z}_t} \mathrm{d}\Phi(z)}_{\text{savings by unproductive}} + \underbrace{\beta(1-\lambda) W_t \int_{\bar{z}_t}^{\infty} \mathrm{d}\Phi(z)}_{\text{borrowings by productive}}$$
$$= \beta W_t (\lambda \Phi(\bar{z}_t) + 1 - \lambda)$$

Financial accelerator

- Our model produces financial accelerator
- When land price $P_t \uparrow$, current wealth

$$W_t = F(K_t, 1) + P_t \uparrow$$

- When current wealth $W_t \uparrow$, next period capital

$$K_{t+1} = \beta\lambda W_t \int_{\bar{z}_t}^{\infty} z \, \mathrm{d}\Phi(z) \uparrow$$

  and also $W_{t+1} \uparrow$
- Increased wealth feeds back to land price:

$$P_{t+1} = \beta W_{t+1}(\lambda\Phi(\bar{z}_{t+1}) + 1 - \lambda) \uparrow$$

Asymptotic behavior

- Asymptotic behavior of model depends on leverage λ
  - High $\lambda \to$ high investment $\to$ economy grows
  - Low $\lambda \to$ low investment $\to$ economy converges
- Suppose economy asymptotically grows at rate $G > 1$, so

$$K_t \sim kG^t, \quad W_t \sim wG^t, \quad P_t \sim pG^t$$

Elasticity of substitution and land rents

- Let σ be asymptotic elasticity of substitution between capital and land:

$$\sigma = -\lim_{K\to\infty} \frac{\partial \log(K/X)}{\partial \log(F_K/F_X)}$$

- Recalling $F_K/K \to m > 0$, can show $\sigma > 1$ regardless of F
- Then land rent $r_t \sim rG^{t/\sigma}$ grows at rate $G^{1/\sigma} < G$
- Risk-free rate converges to

$$R_t = \frac{P_{t+1} + r_{t+1}}{P_t} \to G$$

- Hence productivity threshold converges to

$$\bar{z}_t = \frac{R_t}{F_K(K_{t+1}, 1)} \to \frac{G}{m}$$

Growth condition

- Letting $t \to \infty$ in equilibrium conditions, get long-run growth condition

$$G/m = \frac{\beta\lambda \int_{G/m}^{\infty} z \, \mathrm{d}\Phi(z)}{1 - \beta(\lambda\Phi(G/m) + 1 - \lambda)}.$$

- Setting $G = 1$ and solving for λ, we get $G > 1$ if and only if

$$\lambda > \bar{\lambda} := \frac{1-\beta}{\beta} \frac{1}{\int_{1/m}^{\infty} (mz - 1) \, \mathrm{d}\Phi(z)}.$$

Existence of equilibrium

Proposition

Let Z be random variable with cdf Φ and assume $\beta \, \mathrm{E}[mZ \mid mZ \geq 1] > 1$. Let $\bar{\lambda}$ be leverage threshold as above. Then

1. *If $\lambda < \bar{\lambda}$, there exists a unique equilibrium converging to steady state.*
2. *If $\lambda > \bar{\lambda}$, there exists a unique equilibrium with order of magnitude $K_t \sim kG^t$, $W_t \sim wG^t$, $P_t \sim pG^t$, where $G > 1$ satisfies long-run growth condition.*

Fundamental value and land bubble

- Model features no aggregate uncertainty
- Let $q_t := 1/\prod_{s=0}^{t-1} R_s$ be date-0 price of consumption delivered at time t (price of zero-coupon bond with maturity t)
- By definition, fundamental value of land is present value of land rents

$$V_t := \frac{1}{q_t} \sum_{s=t+1}^{\infty} q_s r_s,$$

- We say land price exhibits bubble if $P_t > V_t$

Unbalanced growth and asset bubble

- With growth $G > 1$, we know $R_t \sim G$ and $r_t \sim G^{t/\sigma}$ (with $\sigma > 1$)
- Thus, growth rate of economy (G) exceeds growth rate of rents ($G^{1/\sigma}$): unbalanced growth
- Under unbalanced growth, fundamental value of land

$$V_t \sim \sum_{s=t+1}^{\infty} G^{t-s} r_s \sim G^{t/\sigma}$$

- But with growth, we know $P_t \sim G^t > G^{t/\sigma} \sim V_t$, so land necessarily exhibits bubble
  - Asset bubbles are fundamentally nonstationary phenomena (Hirano and Toda, 2024a,b)

Land bubble characterization and phase transition

Theorem

Let $\bar{\lambda}$ be leverage threshold. Then

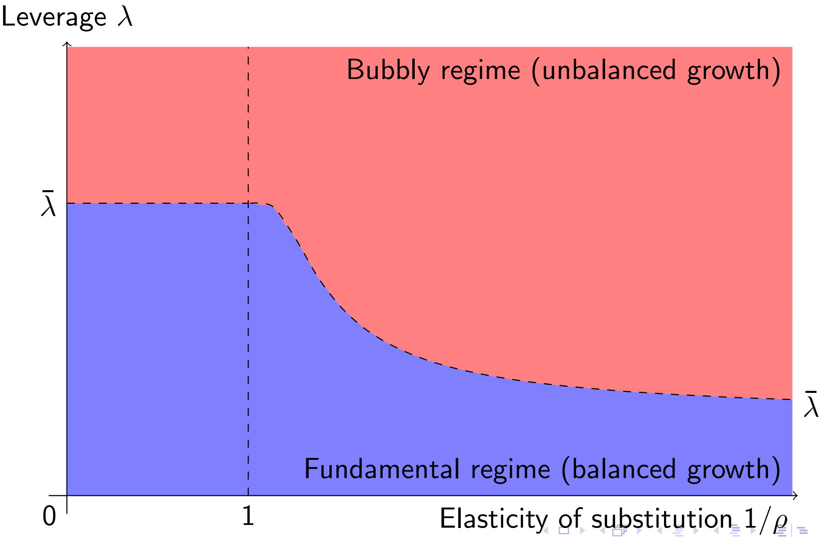
1. *If $\lambda < \bar{\lambda}$, we have $P_t = V_t$ for all t. The economy exhibits balanced growth and the price-rent ratio converges to a positive constant.*
2. *If $\lambda > \bar{\lambda}$, we have $P_t > V_t$ for all t. The economy exhibits unbalanced growth and the price-rent ratio diverges to ∞.*

- Economy exhibits phase transition
  - Low λ: exogenous growth ($G = 1$), fundamental value
  - High λ: endogenous growth ($G > 1$), asset bubble

Numerical example

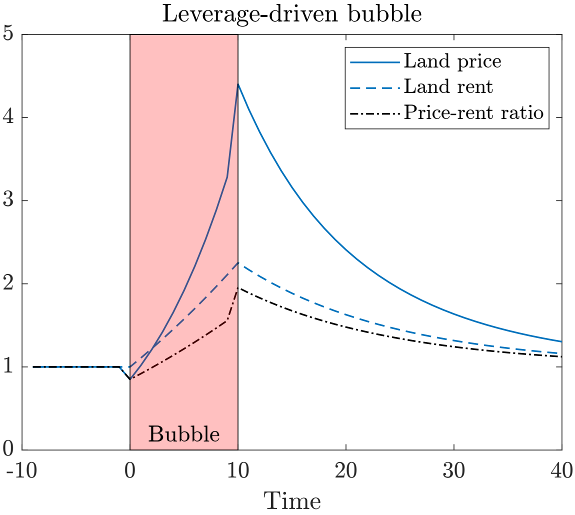
- CES production function
- Exponentially distributed productivities: $\Phi(z) = 1 - \mathrm{e}^{-\gamma z}$
- Parameters: $\beta = 0.95$, $\alpha = 0.5$, $\delta = 0.08$, $\gamma = -\log(0.1)$ (so $\Pr(z > 1) = 0.1$)
- Vary leverage λ and elasticity of substitution *in production function* $1/\rho$

Phase transition



Leverage-driven bubble

- Set $\rho = 1$ (Cobb-Douglas)
- Change leverage λ from 1 to 2, then back to 1 (unanticipated)



Two-sector model

- Consider special case with linear production function

$$F(K, X) = mK + DX$$

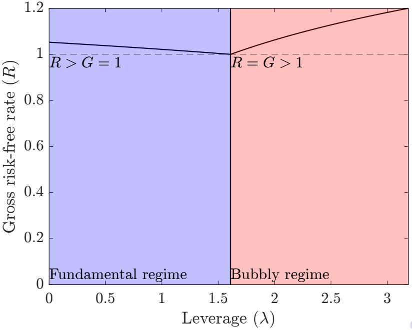
- Can be interpreted as two-sector economy
  1. Modern capital-intensive sector
  2. Traditional land-intensive sector (agriculture)
- For analytical tractability, consider "large open economy"
  - Open economy: interest rate R constant
  - Large economy: external savings/borrowings negligible $\rightarrow$ pins down R
- Can show existence and uniqueness of equilibrium
- Can characterize wealth distribution

V-shaped interest rate

Proposition

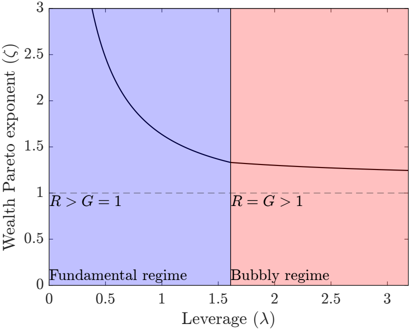
Equilibrium gross risk-free rate R is strictly decreasing (increasing) for $\lambda < \bar{\lambda}$ ($\lambda > \bar{\lambda}$).

- In fundamental regime, as leverage relaxed, marginal investor becomes less productive, hence low interest rate
- In Bubbly regime, as leverage relaxed, $G = R \uparrow$



Bubbles and wealth inequality

- Use Beare and Toda (2022) formula to compute Pareto exponent
- In fundamental regime, as leverage relaxed, unproductive agents earn low $R \to$ higher wealth inequality
- In bubbly regime, as leverage relaxed, $R \uparrow$ and unproductive catch up



Concluding remarks

- Proposed macro-finance theory to think about bubbles attached to dividend-paying assets
  - Model circumvents shortcomings of pure bubble models including lack of realism, equilibrium indeterminacy, and inability to connect to empirical literature
- Balanced or unbalanced growth endogenously determined by financial leverage:
  - Low leverage: weak financial accelerator, balanced growth, fundamental value
  - High leverage: strong financial accelerator, unbalanced growth, bubble
- Hence with financial (or technological) development and presence of fixed factor like land, bubbles become inevitable

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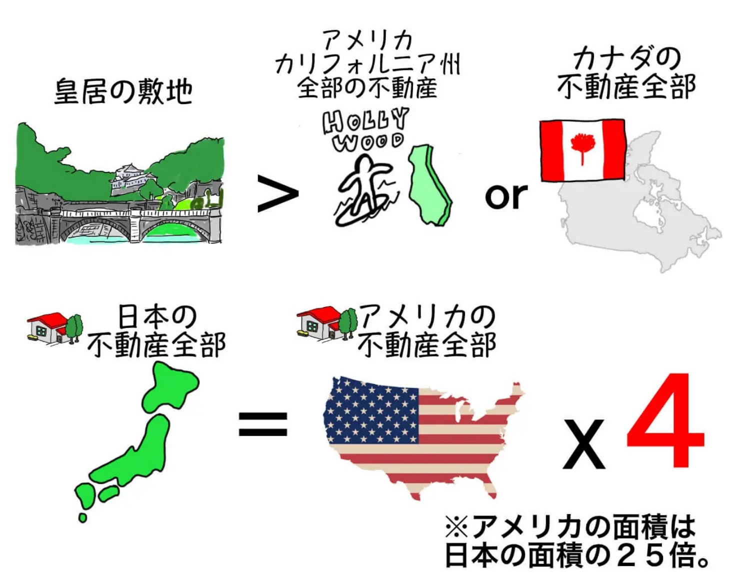
Figure: Japanese bubble in late 1980s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