Unbalanced Growth, Elasticity of Substitution, and Land Overvaluation

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Seminar @UVA
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Research agenda on bubbles

- Research topic I am currently most excited about is asset price bubbles
  - Definition: asset price $P >$ fundamental value $V =$ present value of dividends D
- Several working papers
  1. "Bubble Necessity Theorem" (Hirano and Toda, 2023c), R&R JPE
  2. "Leverage, Endogenous Unbalanced Growth, and Asset Price Bubbles" (Hirano, Jinnai, and Toda, 2022)
  3. "Unbalanced Growth, Elasticity of Substitution, and Land Overvaluation" (Hirano and Toda, 2023d), today
  4. "A Theory of Rational Housing Bubbles with Phase Transitions" (Hirano and Toda, 2023a)
  5. "Bubble Economics" (Hirano and Toda, 2023b), review article to be published in 50th year anniversary issue of JME

“Bubble Necessity Theorem”

- Prove Bubble Characterization Lemma (next slide)
- Construct large class of plausible economic models such that
  - bubbly equilibrium exists
  - no fundamental equilibria exist
- Hence bubbles are necessity or inevitable in some models
- Bubble necessity condition:

$$R < G_d < G,$$

  where G: economic growth rate, G_d: dividend growth rate, R: counterfactual autarky interest rate
- Condition $G_d < G$ naturally arises with multiple sectors with heterogeneous productivity growth

Definition of bubbles

- Asset dividend $D_t \geq 0$, price $P_t \geq 0$ at $t = 0, 1, \ldots$
- With Arrow-Debreu (date-0) price $q_t > 0$, no-arbitrage implies

$$q_t P_t = q_{t+1}(P_{t+1} + D_{t+1}), \quad \text{so}$$

$$P_0 = \sum_{t=1}^{T} q_t D_t + q_T P_T \quad \text{by iteration}$$

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- Letting $T \to \infty$, get

$$P_0 = \underbrace{\sum_{t=1}^{\infty} q_t D_t}_{=: V_0 = \text{fundamental value}} + \underbrace{\lim_{T\to\infty} q_T P_T}_{\text{bubble component}}$$

- If $\lim_{T\to\infty} q_T P_T = 0$, transversality condition holds and no bubble; if > 0, bubble

Bubble Characterization Lemma

Lemma

If $P_t > 0$ for all t, asset price exhibits bubble if and only if

$$\sum_{t=1}^{\infty} \frac{D_t}{P_t} < \infty$$

- Hence bubble if and only if sum of dividend yields finite
- Except pure bubble models ($D_t \equiv 0$), bubbles are fundamentally nonstationary phenomena: price must grow faster than dividend

Proof

- By no-arbitrage,

$$q_{t-1}P_{t-1} = q_t(P_t + D_t) \iff \frac{q_{t-1}P_{t-1}}{q_tP_t} = 1 + \frac{D_t}{P_t}$$

- Taking product from $t = 1$ to $t = T$, get

$$\frac{q_0P_0}{q_TP_T} = \prod_{t=1}^{T}\left(1 + \frac{D_t}{P_t}\right)$$

- Expanding terms and using $1 + x \leq \mathrm{e}^x$, we obtain

$$1 + \sum_{t=1}^{T}\frac{D_t}{P_t} \leq \frac{q_0P_0}{q_TP_T} \leq \exp\left(\sum_{t=1}^{T}\frac{D_t}{P_t}\right)$$

- Let $T \to \infty$ and use definition of TVC

"Leverage, Endogenous Unbalanced Growth, and Asset Price Bubbles"

- Production uses capital and land as inputs
- Agents subject to idiosyncratic investment risk
- Productive agents borrow using leverage
- Phase transition as leverage relaxed
  - With low leverage, economy converges to steady state, land price reflects fundamentals
  - With high leverage, no steady state and economy grows endogenously; land bubble necessarily emerges
- Financial development and bubbles tightly linked; discussion of Japanese economy in 1980s

"A Theory of Rational Housing Bubbles with Phase Transitions"

- Standard two-period OLG model but with housing
- Agents demand housing service when turning from young to old
- Phase transition as young's income share rises
  - With low income, housing price reflects fundamentals
  - With intermediate income, coexistence of fundamental and bubbly equilibria
  - With high income, housing bubble necessarily emerges
- Same effect if young given more credit
- Brief empirical application consistent with theory

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Land as factor of production

- As economies develop and per capita income $\uparrow$, importance of land as factor of production $\downarrow$
- One reason could be humans face biological (quantity) constraints
  - Food intake limited (land produces agricultural products)
  - Leisure time limited (land produces amenities like tennis courts and national parks)

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- One reason could be humans face biological (quantity) constraints
  - Food intake limited (land produces agricultural products)
  - Leisure time limited (land produces amenities like tennis courts and national parks)
- Another could be difference in productivity growth
- Think about quality improvement in
  - "land-intensive products" (e.g., dining, housing, outdoor experience)
  - "high-tech stuff" (e.g., Internet, smart phones, electric vehicles)

GDP share of agriculture decreases with income

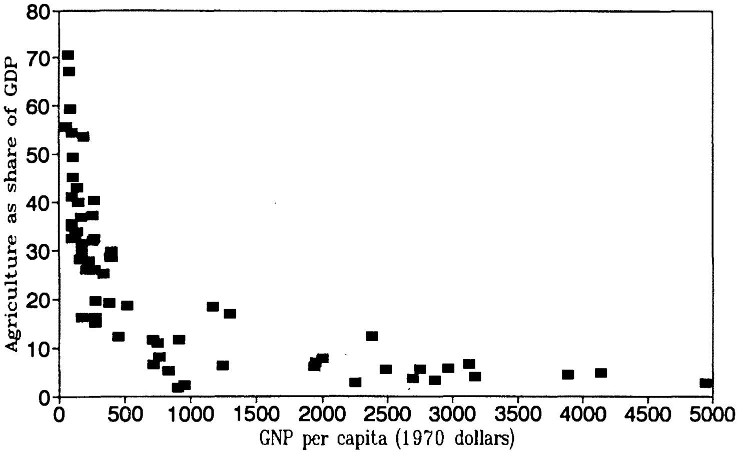


Figure: Echevarria (1997, Figure 2)

Employment share of agriculture decreases over time

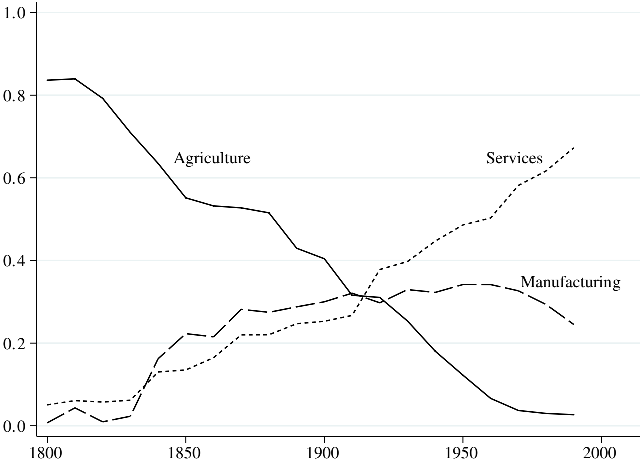


FIGURE 20.1 The share of U.S. employment in agriculture, manufacturing, and services, 1800–2000.

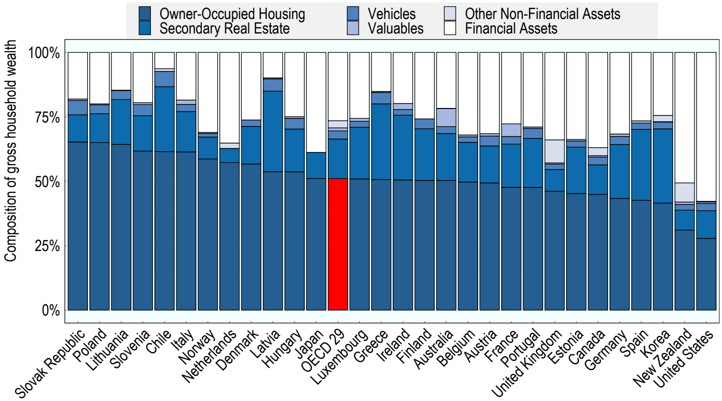
Figure: Acemoglu (2009, Figure 20-1)

Land as store of value

- Land continues to play significant role as store of value
- In many countries, housing wealth is substantial

Figure 2.1. Average decomposition of household assets, 29 OECD countries

2019 or latest available year



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   - Compare to fiat money and public debt

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4. Non-reproducible
   - Compare to fiat money
5. Property rights well defined
   - Compare to gold, silver

This paper

- Study long-run behavior of land prices in modern economies
  - Importance of land as factor of production $\downarrow$
  - Importance of land as store of value $\rightarrow$
- Main result: Land Overvaluation Theorem

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  Unbalanced growth

  (Productivity growth non-land sector $>$ land sector)

  $+$ Condition on factor elasticity of substitution

  $\Longrightarrow$ Land price bubble

- Land bubbles are
  - ✗ short-run phenomena of boom-bust cycles
  - ✓ long-run phenomena along economic development

Related literature

- OLG model with land McCallum (1987), Mountford (2004)
- Unbalanced growth Baumol (1967), Hansen and Prescott (2002)
- Land/housing bubble Kocherlakota (2013)
- Necessity of bubbles Hirano and Toda (2023c)

Two-sector growth economy with land

- Two-period OLG model (young & old, constant population)
- Cobb-Douglas utility $(1-\beta)\log y_t + \beta \log z_{t+1}$
- Young have labor 1, old 0
- Initial old own land (unit supply, durable, non-reproducible)
- Two sectors with neoclassical production functions

$$F_{1t}(H, X) = A_{1t} H,$$
$$F_{2t}(H, X) = A_{2t} H^{\alpha} X^{1-\alpha},$$

  where H: labor/human capital, X: land
  - Sector 1: labor-intensive (service, finance, information, etc.)
  - Sector 2: land-intensive (agriculture, construction, etc.)
- Productivity $\{(A_{1t}, A_{2t})\}_{t=0}^{\infty}$ exogenous and deterministic (for now)

Equilibrium

- Equilibrium is sequence

$$\{(P_t, r_t, w_t, x_t, y_t, z_t, H_{1t}, H_{2t})\}_{t=0}^{\infty},$$

where P_t: land price, r_t: land rent, w_t: wage, x_t: land holdings, (y_t, z_t): young and old consumption, (H_{1t}, H_{2t}): labor input

- Utility/profit maximization
- Market clearing
  - good
  - land
  - labor

Profit maximization

- Firm j maximizes profit

$$F_{jt}(H, X) - w_t H - r_t X$$

- Assume both sectors active (easy to provide sufficient condition)
- Using $X = 1$, profit maximization is

$$\alpha A_{2t} H_{2t}^{\alpha-1} = w_t = A_{1t} \iff H_{2t} = \alpha^{\frac{1}{1-\alpha}} (A_{2t}/A_{1t})^{\frac{1}{1-\alpha}}$$

- Wage and rent:

$$\begin{aligned} w_t &= A_{1t}, \\ r_t &= (1-\alpha) A_{2t} H_{2t}^{\alpha} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} (A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}} \end{aligned}$$

Utility maximization

- Young maximize utility subject to budget constraints

$$\begin{aligned} \text{Young:} \quad & y_t + P_t x_t = w_t, \\ \text{Old:} \quad & z_{t+1} = (P_{t+1} + r_{t+1}) x_t \end{aligned}$$

- Combine sequential budget constraints to

$$y_t + \frac{1}{R_t} z_{t+1} = w_t,$$

  where $R_t := (P_{t+1} + r_{t+1})/P_t$ is gross return on land

- Because utility Cobb-Douglas, demand is $y_t = (1 - \beta) w_t$

Equilibrium land price

- Because old exit economy, land market clearing implies $x_t = 1$
- Hence equilibrium land price driven by income:

$$P_t = P_t x_t = w_t - y_t = \beta w_t = \beta A_{1t}$$

- Hence rent yield (rent-price ratio) is

$$\frac{r_t}{P_t} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}}}{\beta A_{1t}} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\beta}(A_{2t}/A_{1t})^{\frac{1}{1-\alpha}}$$

- Suppose labor productivity grows faster than land productivity (unbalanced growth, e.g., $A_{1t}/A_{2t} \sim G^t$ with $G > 1$)
- Then $\{r_t/P_t\}$ summable, and land bubble necessarily emerges by Bubble Characterization Lemma ?

Intuition

- Suppose for simplicity that $A_{1t} = G^t$, $A_{2t} = 1$
- Then rent $r_t = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}} \sim G^{-\frac{\alpha t}{1-\alpha}}$
- Land price $P_t = \beta A_{1t} \sim G^t$
- Hence interest rate

$$R_t = \frac{P_{t+1} + r_{t+1}}{P_t} \sim G > 1$$

- Hence fundamental value of land finite, while land price grows exponentially driven by demand for savings, generating land bubble

General case

- Previous example is just illustrative example
- We now consider general stochastic two-period OLG model
- Uncertainty resolved according to filtration $\{\mathcal{F}_t\}_{t=0}^{\infty}$ over probability space $(\Omega, \mathcal{F}, P)$
- Cobb-Douglas utility $(1-\beta)\log y_t + \beta \operatorname{E}_t[\log z_{t+1}]$
- Aggregate production function

$$F_t(H, X) := F(A_{Ht}H, A_{Xt}X),$$

  where
  - F is neoclassical (concave, constant returns to scale)
  - Productivity $\{(A_{Ht}, A_{Xt})\}_{t=0}^{\infty}$ is adapted process
- Note: can always define aggregate production function

Definition of equilibrium

- Equilibrium notion is competitive equilibrium with sequential trading

Definition

A competitive equilibrium consists of adapted processes of prices $\{(P_t, r_t, w_t)\}_{t=0}^{\infty}$, allocations $\{(x_t, y_t, z_t)\}_{t=0}^{\infty}$, and factor inputs $\{(H_t, X_t)\}_{t=0}^{\infty}$ such that,

1. (Utility maximization) (x_t, y_t, z_{t+1}) maximizes utility subject to budget constraints,
2. (Profit maximization) (H_t, X_t) maximizes profit $F_t(H_t, X_t) - w_t H_t - r_t X_t$,
3. (Market clearing) $H_t = 1$, $X_t = 1 = x_t$, and $y_t + z_t = F_t(H_t, X_t)$.

Characterization of equilibrium

Proposition

Economy has unique equilibrium, which is characterized by the following equations:

Wage:	$w_t = F_H(A_{Ht}, A_{Xt})A_{Ht},$
Rent:	$r_t = F_X(A_{Ht}, A_{Xt})A_{Xt},$
Land price:	$P_t = \beta w_t,$
Young consumption:	$y_t = (1-\beta)w_t,$
Old consumption:	$z_t = \beta w_t + r_t$

Elasticity of substitution

- It turns out that elasticity of substitution (ES) is important
- Recall ES defined by change in relative factor inputs with respect to change in relative factor prices

$$\sigma = -\frac{\partial \log(H/X)}{\partial \log(w/r)}$$

- For neoclassical production function, can show ES is

$$\sigma_F(H,X) = \frac{F_H F_X}{F F_{HX}}$$

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Assumption

Elasticity of substitution of neoclassical production function F exceeds 1 at high input levels:

$$\liminf_{H \to \infty} \sigma_F(H, 1) > \sigma > 1.$$

Defending $\sigma_F > 1$ at high input level, I

- Epple, Gordon, and Sieg (2010) use duality to estimate ES between land and non-land factors for producing real estate
  - Micro data from Allegheny County, Pennsylvania
  - $\sigma_F = 1.16$ for residential properties
  - $\sigma_F = 1.39$ for commercial properties
- Ahlfeldt and McMillen (2014) argue EGS approach is robust
  - Find $\sigma_F = 1.25$ for Chicago and Berlin

Defending $\sigma_F > 1$ at high input level, II

- With $\sigma_F < 1$ and unbalanced growth, economy is pathological
- To see why, assume CES production function

$$F_t(H, X) = \left(\alpha(A_{Ht}H)^{1-\rho} + (1-\alpha)(A_{Xt}X)^{1-\rho}\right)^{\frac{1}{1-\rho}},$$

where $\rho = 1/\sigma > 1$

- Assume $(A_{Ht}, A_{Xt}) = (G_H^t, G_X^t)$ with $G_H > G_X$
- Then easy to show

$$R_t = \frac{\beta w_{t+1} + r_{t+1}}{\beta w_t} \to \infty,$$

which is pathological and counterfactual

Defending $\sigma_F > 1$ at high input level, III

Lemma

If F neoclassical with $\lim_{H\to\infty} F_H(H,1) = m > 0$*, then*

$$\liminf_{H\to\infty} \sigma_F(H,1) \geq 1.$$

Defending $\sigma_F > 1$ at high input level, III

Lemma

If F neoclassical with $\lim_{H\to\infty} F_H(H,1) = m > 0$, then

$$\liminf_{H\to\infty} \sigma_F(H,1) \geq 1.$$

- Lemma implies that, if non-land factors don't fully depreciate, then $\sigma_F \geq 1$ always at high input level
- Example: if F CES with partial depreciation

$$F(H,X) = A\left(\alpha H^{1-\rho} + (1-\alpha)X^{1-\rho}\right)^{\frac{1}{1-\rho}} + BH,$$

can show

$$\lim_{H\to\infty} \sigma_F(H,1) = \begin{cases} 1/\rho & \text{if } \rho < 1, \\ 1/\alpha & \text{if } \rho = 1, \\ \infty & \text{if } \rho > 1 \end{cases}$$

Unbalanced growth and land overvaluation

Theorem (Land Overvaluation)

Let F be neoclassical with $\liminf_{H\to\infty}\sigma_F(H,1)>\sigma>1$*. If*

$$\mathrm{E}_0\sum_{t=0}^{\infty}(A_{Ht}/A_{Xt})^{1/\sigma-1}<\infty$$

almost surely, then land is overvalued ($P>V$) in equilibrium.

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almost surely, then land is overvalued ($P > V$) in equilibrium.

Idea of proof.

1. Derive SDF and bound fundamental value V_t from above
2. Use $\sigma > 1$ and summability condition to show $V_t/P_t \to 0$
3. Hence $P_t > V_t$ for large enough t, and also true for all t by backward induction argument □

Two-sector example is special case

- Consider previous example with $F_{1t}(H, X) = A_{1t}H$ and $F_{2t}(H, X) = A_{2t}H^{\alpha}X^{1-\alpha}$
- Aggregate production function is

$$F_t(H, X) := \max\left\{\sum_{j=1}^{2} F_{jt}(H_j, X_j) : \sum_{j=1}^{2} H_j = H, \sum_{j=1}^{2} X_j = X\right\}$$

- After some algebra, can show

$$F_t(H, X) = A_{1t}H + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}}X,$$

- Hence can define $F(H, X) = H + X$ (linear, $\sigma = \infty$) and A_{Ht}, A_{Xt} appropriately to apply Land Overvaluation Theorem

Implications of Land Overvaluation Theorem

1. Elasticity of substitution is crucial for overvaluation
    - Previously unknown
2. Unbalanced growth (nonstationarity) is crucial for overvaluation
    - Economists trained and accustomed to study balanced growth, so asset price bubbles overlooked
    - By Bubble Characterization Lemma ?, only stationary model consistent with bubbles is pure bubble model ($D_t \equiv 0$)
    - Pure bubble model inadequate to study land and housing bubbles ($D_t > 0$)
3. In model, land price fluctuates with productivity, but always bubble (bubbles expand and shrink)

Recurrent stochastic fluctuations

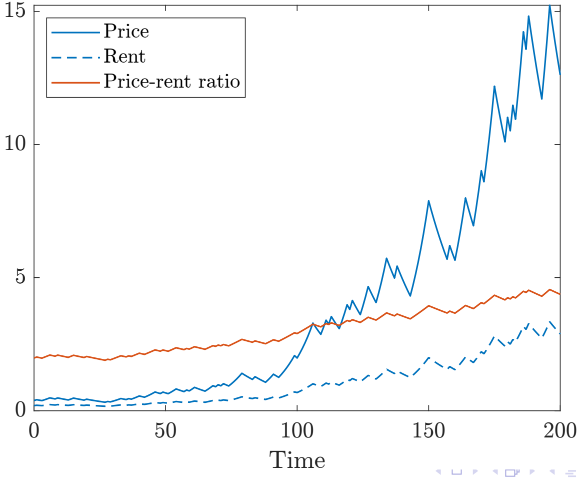
- As example, assume CES production function with $\sigma > 1$ and let $A_t = A_{Ht}/A_{Xt}$ be relative productivity
- Assume $A_t = G_t A_{t-1}$, where $G_t = G_{nn'}$ conditional on transitioning from state n to n' (hidden Markov process)
- Can use dynamic programming argument to check assumption of Land Overvaluation Theorem

Proposition

Let everything be as above and $K = (\pi_{nn'} G_{nn'}^{1/\sigma - 1})$. Then land is overvalued if the spectral radius of K is less than 1.

Numerical example

- Set $\beta = 0.5$, $\alpha = 0.8$, $\sigma = 1.25$, $N = 2$, $\pi_{nn'} = 1/3$ if $n \neq n'$, and $(G_{1n'}, G_{2n'}) = (1.1, 0.95)$ for all n'



Concluding remarks

- Studied long-run behavior of land prices in modern economy (transition from land-intensive to labor/knowledge-intensive)
- Surprising link between unbalanced growth, elasticity of substitution, and land overvaluation
- Messages from our research agenda
    - Bubbles are fundamentally nonstationary phenomena connected to unbalanced growth
    - Bubbles attached to dividend-paying assets under-explored—unlimited potential for applications
    - Bubbles are inevitable in modern economies: policy should focus on management, not prevention

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