Unbalanced Growth and Land Overvaluation

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Land as factor of production

- As economies develop and per capita income $\uparrow$, importance of land as factor of production $\downarrow$
- One reason could be humans face biological (quantity) constraints
  - Food intake limited (land produces agricultural products)
  - Leisure time limited (land produces amenities like tennis courts and national parks)

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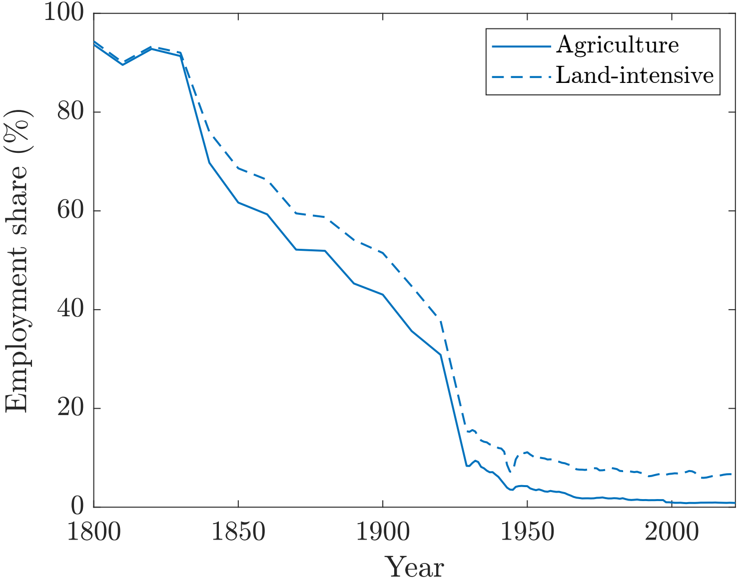
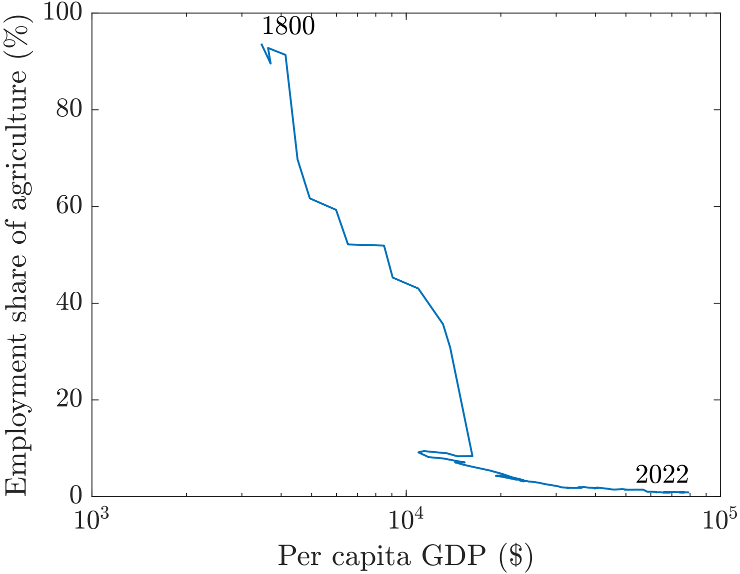
- One reason could be humans face biological (quantity) constraints
    - Food intake limited (land produces agricultural products)
    - Leisure time limited (land produces amenities like tennis courts and national parks)
- Another could be difference in productivity growth
- Think about quality improvement in
    - "land-intensive products" (e.g., dining, housing, outdoor experience)
    - "high-tech stuff" (e.g., Internet, smart phones, electric vehicles)

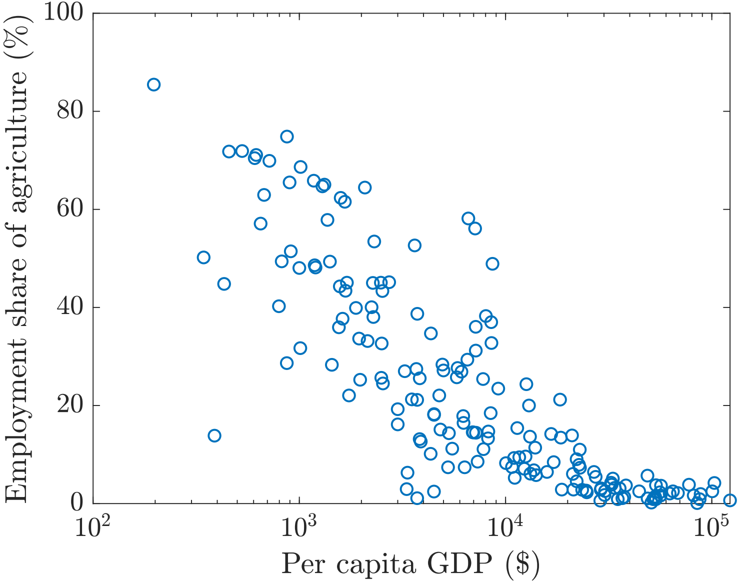
Employment share of agriculture decreases over time



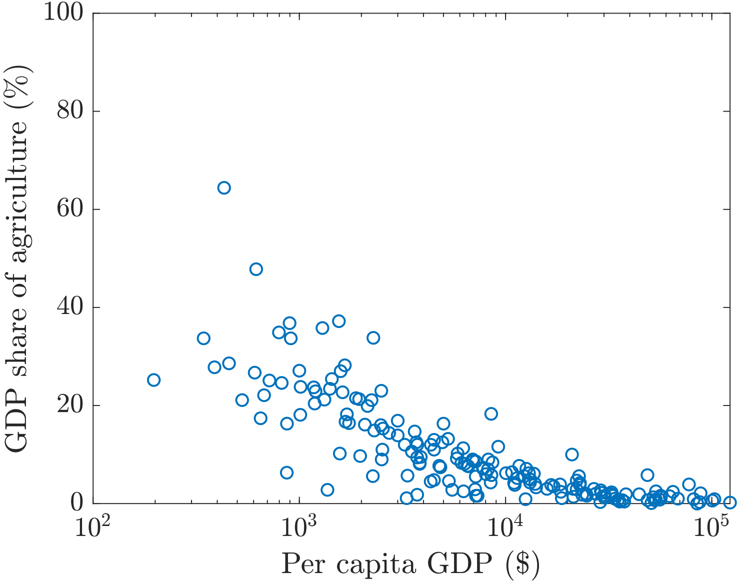
...and along economic development



Same holds across countries

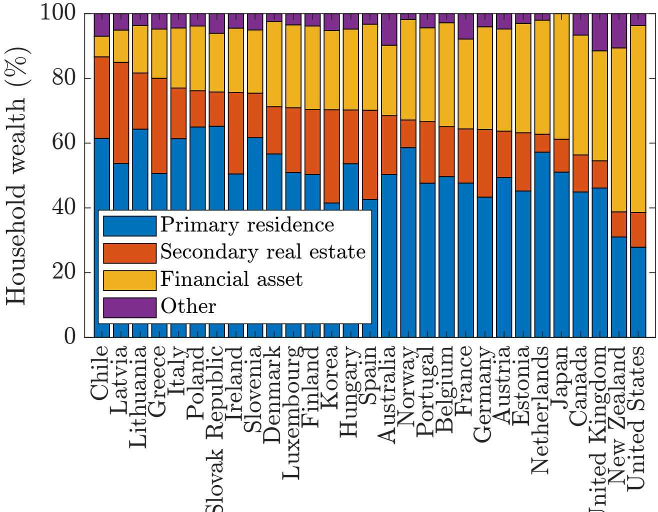


. . . and for GDP share of agriculture



Land as store of value

- Land continues to play significant role as store of value
- In many countries, housing wealth is substantial



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   - Compare to fiat money and public debt

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4. Non-reproducible
    - Compare to fiat money
5. Property rights well defined
    - Compare to gold, silver

This paper

- Study long-run behavior of land prices in modern economies
  - Importance of land as factor of production $\downarrow$
  - Importance of land as store of value $\rightarrow$
- Main result: Land Overvaluation Theorem

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  Unbalanced growth

  (Productivity growth non-land sector $>$ land sector)

  $+$ Condition on factor elasticity of substitution

  $\implies$ Land price bubble

- Land bubbles are
  - ✗ short-run phenomena of boom-bust cycles
  - ✓ long-run phenomena along economic development

Related literature

- OLG model with land McCallum (1987), Rhee (1991), Mountford (2004)
- Unbalanced growth Baumol (1967), Hansen and Prescott (2002), Fujiwara and Matsuyama (2024)
- Land/housing bubble Kocherlakota (2013), Hirano and Toda (2023)
- Necessity of bubbles Hirano and Toda (2024b)
- Bubbles attached to dividend-paying assets Wilson (1981), Tirole (1985), Hirano, Jinnai, and Toda (2022)
- Introduction to rational bubbles Hirano and Toda (2024a,c)

Two-sector growth economy with land

- Two-period OLG model (young & old, constant population)
- Cobb-Douglas utility $(1-\beta)\log c_t^y + \beta \log c_{t+1}^o$
- Young have labor 1, old 0
- Initial old own land (unit supply, durable, non-reproducible)
- Two sectors with neoclassical production functions

$$F_{1t}(H,X) = A_{1t}H,$$
$$F_{2t}(H,X) = A_{2t}H^{\alpha}X^{1-\alpha},$$

  where H: labor/human capital, X: land
  - Sector 1: labor-intensive (service, finance, information, etc.)
  - Sector 2: land-intensive (agriculture, construction, etc.)
- Productivity $\{(A_{1t}, A_{2t})\}_{t=0}^{\infty}$ exogenous and deterministic (for now)

Equilibrium

- Equilibrium is sequence

$$\left\{(P_t, r_t, w_t, x_t, c_t^y, c_t^o, H_{1t}, H_{2t})\right\}_{t=0}^{\infty},$$

where P_t: land price, r_t: land rent, w_t: wage, x_t: land holdings, (c_t^y, c_t^o): young and old consumption, (H_{1t}, H_{2t}): labor input

- Utility/profit maximization
- Market clearing
    - good
    - land
    - labor

Profit maximization

- Firm j maximizes profit

$$F_{jt}(H, X) - w_t H - r_t X$$

- Assume both sectors active (easy to provide sufficient condition)
- Using $X = 1$, profit maximization is

$$\alpha A_{2t} H_{2t}^{\alpha-1} = w_t = A_{1t} \iff H_{2t} = \alpha^{\frac{1}{1-\alpha}} (A_{2t}/A_{1t})^{\frac{1}{1-\alpha}}$$

- Wage and rent:

$$\begin{aligned} w_t &= A_{1t}, \\ r_t &= (1-\alpha) A_{2t} H_{2t}^{\alpha} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} (A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}} \end{aligned}$$

Utility maximization

- Young maximize utility subject to budget constraints

$$\begin{aligned} \text{Young:} \quad & c_t^y + P_t x_t = w_t, \\ \text{Old:} \quad & c_{t+1}^o = (P_{t+1} + r_{t+1}) x_t \end{aligned}$$

- Combine sequential budget constraints to

$$c_t^y + \frac{1}{R_t} c_{t+1}^o = w_t,$$

where $R_t := (P_{t+1} + r_{t+1})/P_t$ is gross return on land

- Because utility Cobb-Douglas, demand is $c_t^y = (1 - \beta) w_t$

Equilibrium land price

- Because old exit economy, land market clearing implies $x_t = 1$
- Hence equilibrium land price driven by income:

$$P_t = P_t x_t = w_t - c_t^y = \beta w_t = \beta A_{1t}$$

- Hence rent yield (rent-price ratio) is

$$\frac{r_t}{P_t} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}}}{\beta A_{1t}} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\beta}(A_{2t}/A_{1t})^{\frac{1}{1-\alpha}}$$

- Suppose labor productivity grows faster than land productivity (unbalanced growth, e.g., $A_{1t}/A_{2t} \sim G^t$ with $G > 1$)
- Then $\{r_t/P_t\}$ summable, and land bubble necessarily emerges by Bubble Characterization Lemma ?

Intuition

- Suppose for simplicity that $A_{1t} = G^t$, $A_{2t} = 1$
- Then rent $r_t = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}} \sim G^{-\frac{\alpha t}{1-\alpha}}$
- Land price $P_t = \beta A_{1t} \sim G^t$
- Hence interest rate

$$R_t = \frac{P_{t+1} + r_{t+1}}{P_t} \sim G > 1$$

- Hence fundamental value of land finite, while land price grows exponentially driven by demand for savings, generating land bubble

General case

- Previous example is just illustrative example
- We now consider general stochastic two-period OLG model
- Uncertainty resolved according to filtration $\{\mathcal{F}_t\}_{t=0}^{\infty}$ over probability space $(\Omega, \mathcal{F}, P)$
- Cobb-Douglas utility $(1-\beta)\log c_t^y + \beta\,\mathrm{E}_t[\log c_{t+1}^o]$
- Aggregate production function

$$F_t(H,X) := F(A_{Ht}H, A_{Xt}X),$$

  where
  - F is neoclassical (concave, constant returns to scale)
  - Productivity $\{(A_{Ht}, A_{Xt})\}_{t=0}^{\infty}$ is adapted process
- Note: can always define aggregate production function

Definition of equilibrium

- Equilibrium notion is competitive equilibrium with sequential trading

Definition

A competitive equilibrium consists of adapted processes of prices $\{(P_t, r_t, w_t)\}_{t=0}^{\infty}$, allocations $\left\{(x_t, c_t^y, c_t^o)\right\}_{t=0}^{\infty}$, and factor inputs $\{(H_t, X_t)\}_{t=0}^{\infty}$ such that,

1. (Utility maximization) (x_t, c_t^y, c_{t+1}^o) maximizes utility subject to budget constraints,
2. (Profit maximization) (H_t, X_t) maximizes profit $F_t(H_t, X_t) - w_t H_t - r_t X_t$,
3. (Market clearing) $H_t = 1$, $X_t = 1 = x_t$, and $c_t^y + c_t^o = F_t(H_t, X_t)$.

Characterization of equilibrium

Proposition

Economy has unique equilibrium, which is characterized by the following equations:

Wage:	$w_t = F_H(A_{Ht}, A_{Xt})A_{Ht},$
Rent:	$r_t = F_X(A_{Ht}, A_{Xt})A_{Xt},$
Land price:	$P_t = \beta w_t,$
Young consumption:	$c_t^y = (1-\beta)w_t,$
Old consumption:	$c_t^o = \beta w_t + r_t$

Elasticity of substitution

- It turns out that elasticity of substitution (ES) is important
- Recall ES defined by change in relative factor inputs with respect to change in relative factor prices

$$\sigma = -\frac{\partial \log(H/X)}{\partial \log(w/r)}$$

- For neoclassical production function, can show ES is

$$\sigma_F(H, X) = \frac{F_H F_X}{F F_{HX}}$$

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Assumption

Elasticity of substitution of neoclassical production function F exceeds 1 at high input levels:

$$\liminf_{H\to\infty} \sigma_F(H, 1) > \sigma > 1.$$

Defending $\sigma_F > 1$ at high input level, I

- Epple, Gordon, and Sieg (2010) use duality to estimate ES between land and non-land factors for producing real estate
  - Micro data from Allegheny County, Pennsylvania
  - $\sigma_F = 1.16$ for residential properties
  - $\sigma_F = 1.39$ for commercial properties
- Ahlfeldt and McMillen (2014) argue EGS approach is robust
  - Find $\sigma_F = 1.25$ for Chicago and Berlin

Defending $\sigma_F > 1$ at high input level, II

- With $\sigma_F < 1$ and unbalanced growth, economy is pathological
- To see why, assume CES production function

$$F_t(H,X) = \left(\alpha(A_{Ht}H)^{1-\rho} + (1-\alpha)(A_{Xt}X)^{1-\rho}\right)^{\frac{1}{1-\rho}},$$

where $\rho = 1/\sigma > 1$

- Assume $(A_{Ht}, A_{Xt}) = (G_H^t, G_X^t)$ with $G_H > G_X$
- Then easy to show

$$R_t = \frac{\beta w_{t+1} + r_{t+1}}{\beta w_t} \to \infty,$$

which is pathological and counterfactual

Defending $\sigma_F > 1$ at high input level, III

Lemma

If F neoclassical with $\lim_{H\to\infty} F_H(H,1) = m > 0$, *then*

$$\liminf_{H\to\infty} \sigma_F(H,1) \geq 1.$$

Defending $\sigma_F > 1$ at high input level, III

Lemma

If F neoclassical with $\lim_{H\to\infty} F_H(H,1) = m > 0$, then

$$\liminf_{H\to\infty} \sigma_F(H,1) \geq 1.$$

- Lemma implies that, if non-land factors don't fully depreciate, then $\sigma_F \geq 1$ always at high input level
- Example: if F CES with partial depreciation

$$F(H,X) = A\left(\alpha H^{1-\rho} + (1-\alpha)X^{1-\rho}\right)^{\frac{1}{1-\rho}} + BH,$$

can show

$$\lim_{H\to\infty} \sigma_F(H,1) = \begin{cases} 1/\rho & \text{if } \rho < 1, \\ 1/\alpha & \text{if } \rho = 1, \\ \infty & \text{if } \rho > 1 \end{cases}$$

Unbalanced growth and land overvaluation

Theorem (Land Overvaluation)

Let F be neoclassical with $\liminf_{H\to\infty} \sigma_F(H,1) > \sigma > 1$. *If*

$$\mathrm{E}_0 \sum_{t=0}^{\infty} (A_{Ht}/A_{Xt})^{1/\sigma-1} < \infty$$

almost surely, then land is overvalued ($P > V$) in equilibrium.

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Idea of proof.

1. Derive SDF and bound fundamental value V_t from above
2. Use $\sigma > 1$ and summability condition to show $V_t/P_t \to 0$
3. Hence $P_t > V_t$ for large enough t, and also true for all t by backward induction argument □

Two-sector example is special case

- Consider previous example with $F_{1t}(H, X) = A_{1t}H$ and $F_{2t}(H, X) = A_{2t}H^{\alpha}X^{1-\alpha}$
- Aggregate production function is

$$F_t(H, X) := \max \left\{ \sum_{j=1}^{2} F_{jt}(H_j, X_j) : \sum_{j=1}^{2} H_j = H, \sum_{j=1}^{2} X_j = X \right\}$$

- After some algebra, can show

$$F_t(H, X) = A_{1t}H + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(A_{2t}/A_{1t}^{\alpha})^{\frac{1}{1-\alpha}}X,$$

- Hence can define $F(H, X) = H + X$ (linear, $\sigma = \infty$) and A_{Ht}, A_{Xt} appropriately to apply Land Overvaluation Theorem

Implications of Land Overvaluation Theorem

1. Elasticity of substitution is crucial for overvaluation
    - Previously unknown
2. Unbalanced growth (nonstationarity) is crucial for overvaluation
    - Economists trained and accustomed to study balanced growth, so asset price bubbles overlooked
    - By Bubble Characterization Lemma ?, only stationary model consistent with bubbles is pure bubble model ($D_t \equiv 0$)
    - Pure bubble model inadequate to study land and housing bubbles ($D_t > 0$)
3. In model, land price fluctuates with productivity, but always bubble (bubbles expand and shrink)

Recurrent stochastic fluctuations

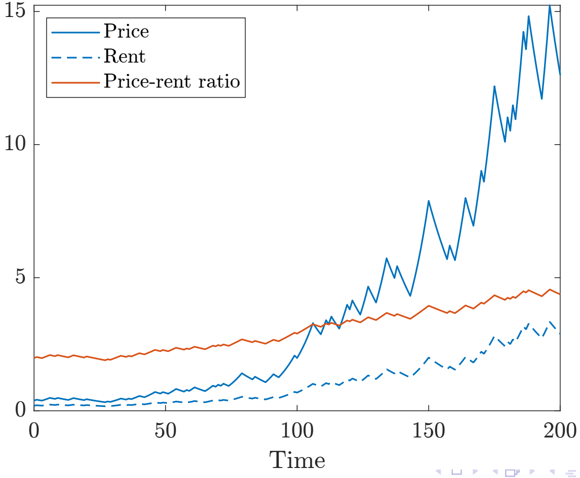
- As example, assume CES production function with $\sigma > 1$ and let $A_t = A_{Ht}/A_{Xt}$ be relative productivity
- Assume $A_t = G_t A_{t-1}$, where $G_t = G_{nn'}$ conditional on transitioning from state n to n' (hidden Markov process)
- Can use dynamic programming argument to check assumption of Land Overvaluation Theorem

Proposition

Let everything be as above and $K = (\pi_{nn'} G_{nn'}^{1/\sigma - 1})$. Then land is overvalued if the spectral radius of K is less than 1.

Numerical example

- Set $\beta = 0.5$, $\alpha = 0.8$, $\sigma = 1.25$, $N = 2$, $\pi_{nn'} = 1/3$ if $n \neq n'$, and $(G_{1n'}, G_{2n'}) = (1.1, 0.95)$ for all n'



Concluding remarks

- Studied long-run behavior of land prices in modern economy (transition from land-intensive to labor/knowledge-intensive)
- Surprising link between unbalanced growth, elasticity of substitution, and land overvaluation
- Messages from our research agenda
  - Bubbles are fundamentally nonstationary phenomena connected to unbalanced growth
  - Bubbles attached to dividend-paying assets under-explored—unlimited potential for applications
  - Bubbles are inevitable in modern economies: policy should focus on management, not prevention

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Definition of bubbles

- Asset dividend $D_t \geq 0$, price $P_t \geq 0$ at $t = 0, 1, \ldots$
- With Arrow-Debreu (date-0) price $q_t > 0$, no-arbitrage implies

$$q_t P_t = q_{t+1}(P_{t+1} + D_{t+1}), \quad \text{so}$$

$$P_0 = \sum_{t=1}^{T} q_t D_t + q_T P_T \quad \text{by iteration}$$

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- Letting $T \to \infty$, get

$$P_0 = \underbrace{\sum_{t=1}^{\infty} q_t D_t}_{=:V_0=\text{fundamental value}} + \underbrace{\lim_{T\to\infty} q_T P_T}_{\text{bubble component}}$$

- If $\lim_{T\to\infty} q_T P_T = 0$, transversality condition holds and no bubble; if > 0, bubble

Bubble Characterization Lemma

Lemma

If $P_t > 0$ for all t, asset price exhibits bubble if and only if

$$\sum_{t=1}^{\infty} \frac{D_t}{P_t} < \infty$$

- This is Proposition 7 of Montrucchio (2004)
- Hence bubble if and only if sum of dividend yields finite
- Except pure bubble models ($D_t \equiv 0$), bubbles are fundamentally nonstationary phenomena: price must grow faster than dividend

Proof

- By no-arbitrage,

$$q_{t-1}P_{t-1} = q_t(P_t + D_t) \iff \frac{q_{t-1}P_{t-1}}{q_t P_t} = 1 + \frac{D_t}{P_t}$$

- Taking product from $t = 1$ to $t = T$, get

$$\frac{q_0 P_0}{q_T P_T} = \prod_{t=1}^{T} \left(1 + \frac{D_t}{P_t}\right)$$

- Expanding terms and using $1 + x \leq \mathrm{e}^x$, we obtain

$$1 + \sum_{t=1}^{T} \frac{D_t}{P_t} \leq \frac{q_0 P_0}{q_T P_T} \leq \exp\left(\sum_{t=1}^{T} \frac{D_t}{P_t}\right)$$

- Let $T \to \infty$ and use definition of TVC